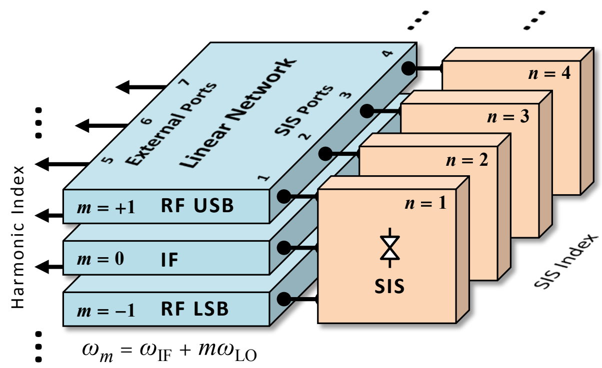
SuperMix SIS Mixer Modeling

Here is a diagram of how SuperMix represents the circuit topology of a multi-junction mixer:



Each block represents a scattering representation of the corresponding object. The blue blocks represent the linear embedding network responses at various harmonic frequencies; the orange blocks represent the various SIS junctions which mix the different harmonic frequencies. The external ports connect to the RF source channel(s) and the IF output channel(s). The scattering representations each include a scattering matrix $\mathbf{S}$ and a noise wave power spectral density correlation matrix $\mathbf{C}$.

For the linear embedding network representations, if their internal noise originates solely from passive, dissipative elements, then a Callen-Welton formula is used for $\mathbf{C}$:

$$\mathbf{C}(\nu) = \frac{h\nu}{2k_\mathrm{B}} \coth\left(\frac{h\nu}{2k_\mathrm{B}T}\right)(\mathbf{I} - \mathbf{S}\mathbf{S}^\dagger)$$

($\mathbf{I}$, of course, is an identity matrix, and SuperMix expresses the elements of $\mathbf{C}$ in temperature units) see Ward et al. (1999). This formula is encoded in the SuperMix library file *supermix/lib-src/sdata.cc*.

In the case of the SIS junctions, the quantum admittance representation presented in Tucker (1979) and Withington and Kollberg (1989) is used. Each SIS current noise correlation matrix $\mathbf{H}$ is calculated using a formula derived from the above references:

$$H_{mn} = \frac{eV_\mathrm{n}}{R_\mathrm{n}} \sum_k C_k\, C^*{}_{k+m-n} \Big\{$$

$$\coth\left[\frac{eV_n}{2k_\mathrm{B}T}\left(\tilde{V}_{bias} + [k+m]\tilde{V}_{LO} + \tilde{V}_{IF}\right)\right] \tilde{I}_{dc}\left(\tilde{V}_{bias} + [k+m]\tilde{V}_{LO} + \tilde{V}_{IF}\right) +$$

$$\coth\left[\frac{eV_n}{2k_\mathrm{B}T}\left(\tilde{V}_{bias} + [k-n]\tilde{V}_{LO} - \tilde{V}_{IF}\right)\right] \tilde{I}_{dc}\left(\tilde{V}_{bias} + [k-n]\tilde{V}_{LO} - \tilde{V}_{IF}\right)$$

$$\Big\}$$

The '~' notation represents normalized SIS I-V values where V_n and R_n are the SIS I-V curve normalizing voltage and resistance. The LO and IF voltages are normalized frequencies $\tilde{V}(\nu) = h\nu / eV_\text{n}$. The $I_{dc}(V)$ is an interpolation of the measured DC SIS I-V characteristic. These equations, of course, give the quantum (Callen-Welton) version of the harmonic shot noise current spectral densities generated by the SIS when pumped by the LO. Withington and Kollberg's complex harmonic coefficients C_k are calculated by SuperMix as part of the mixer's large-signal, harmonic balance calculation of the SIS operating states. The code for the SIS calculations is found in the SuperMix library files *supermix/lib-src/ckdata.cc* and *supermix/lib-src/sisdevice.cc*, and the harmonic balance routine is in *supermix/lib-src/balance.cc*.

With each SIS device's $\mathbf{Y}$ and $\mathbf{H}$ matrices in hand, they are converted to the wave representation's $\mathbf{S}$ and $\mathbf{C}$ using the standard formulas:

$$\mathbf{S} = (\mathbf{I} + z_0\mathbf{Y})^{-1}(\mathbf{I} - z_0\mathbf{Y})$$

$$\mathbf{C} = \frac{z_0}{4k_\text{B}}(\mathbf{I} + \mathbf{S})\,\mathbf{H}\,(\mathbf{I} + \mathbf{S})^\dagger$$

(z_0 is the $\mathbf{S}$ normalizing impedance). These expressions are coded in the SuperMix library file *supermix/lib-src/sdata.cc* and are called by the mixer's small-signal analysis routines found in *supermix/lib-src/analyze.cc*.

Now $\mathbf{S}$ and $\mathbf{C}$ matrix representations are available for each of the elements' blocks shown in the mixer topology figure. The mixer calculation then combines them into a single circuit $\mathbf{S}$ and $\mathbf{C}$ matrix representation using the linear connection method described in chapter 3 of Scott Wedge's Thesis, which also provides the above admittance to wave representation conversion expressions. The method used by the mixer is different from but equivalent to Wedge's "network reduction by subnetwork growth" algorithm used by the SuperMix *circuit* class, but it is optimized for the mixer's particular circuit topology. The linear connection algorithm is encoded in *supermix/lib-src/analyze.cc*. The result is a single $\mathbf{S}$ and $\mathbf{C}$ pair connecting the SIS mixer circuit's IF and RF sideband ports, and is obtained by calling the mixer object's get_data() member function.

The resulting noise correlation matrix does not include the quantum and thermal noise incident from the RF sideband channels and propagated through $\mathbf{S}$ to the mixer's IF output. To get the full quantum noise contribution from the RF channels, use the mixer object's get_term_data() member function to terminate the RF sideband ports and calculate a resulting $\mathbf{S}$ and $\mathbf{C}$ pair which connect only the IF ports. The mixer will then calculate the full quantum noise contribution to its output noise required by Caves and Wengler. The proper programming method wherein SuperMix will calculate the full mixer noise temperature is illustrated in the example program *supermix/examples/mixer/mixer.cc*.