

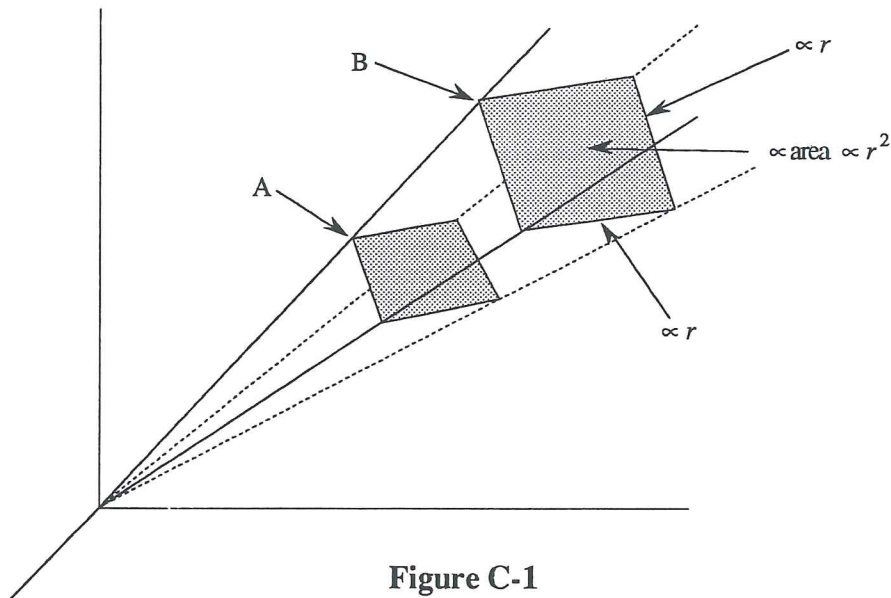
## APPENDIX C

### SOLID ANGLE

The number of scattered particles through a small area  $dA$  is proportional to the area  $dA$ ,

$$dN \propto dA \quad (\text{C-1})$$

In what follows the area  $dA$  is normal to  $r$ , the vector from the scattering center to  $dA$ . Fig. C-1, we hope, shows that number of scattered particles is only a function of  $dA/r^2$ .



**Figure C-1**

The number of scattered particles is the same through areas A and B.

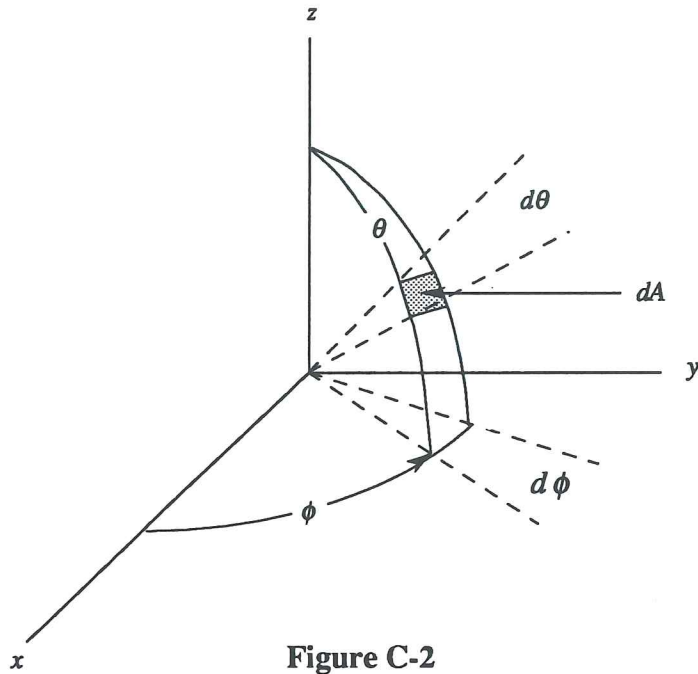
Let's include this factor in Eq. C-1,

$$dN \propto \frac{dA}{r^2} \quad (\text{C-2})$$

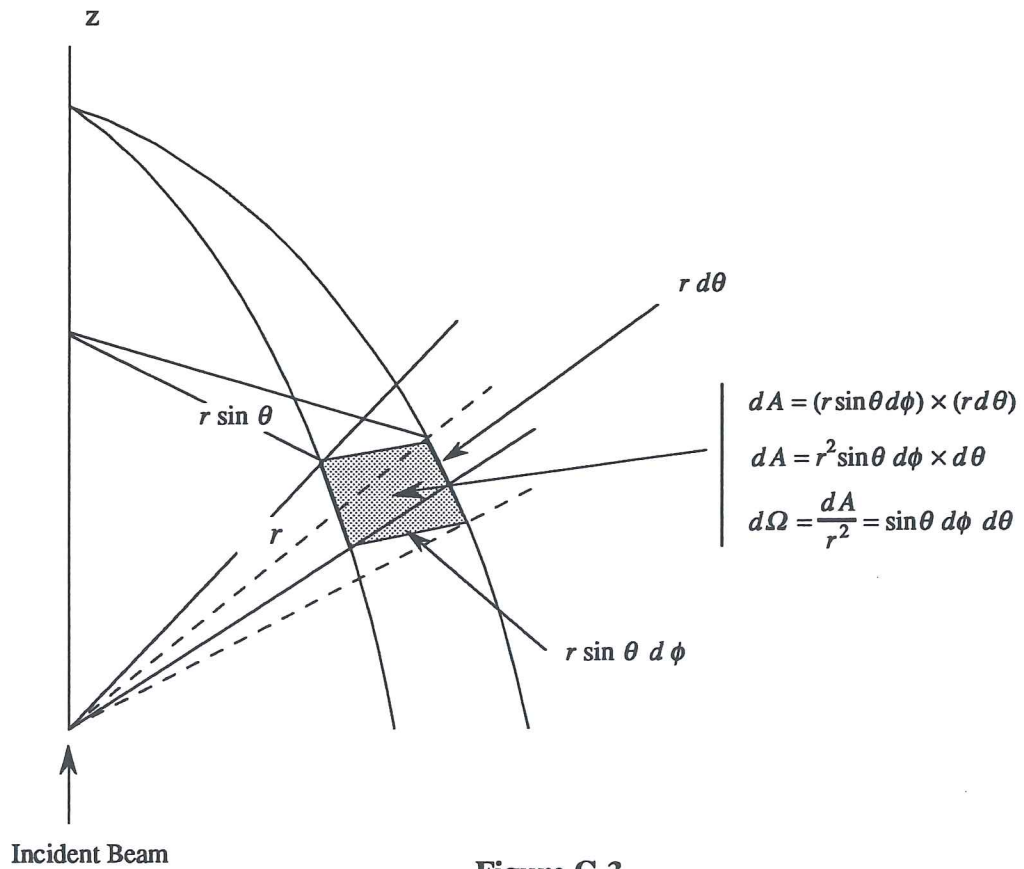
$dA/r^2$  is called the solid angle subtended by  $dA$ , and is usually denoted by  $d\Omega$

$$d\Omega = \frac{dA}{r^2} \quad (\text{C-3})$$

In Figs. C-2 and C-3 we show how to express  $d\Omega$  in spherical coordinates.



**Figure C-2**  
Calculating the solid angle in spherical coordinates.



**Figure C-3**  
Calculating the solid angle in spherical coordinates.

The result of this calculation is

$$d\Omega = \frac{dA}{r^2} = \sin \theta d\phi \times d\theta \quad (\text{C-4})$$

Inserting Eqs. C-3 and C-4 in Eq. C-2 we obtain

$$dN \propto \sin \theta d\phi d\theta \quad (\text{C-5})$$

By definition of the differential scattering cross section, the number of particles scattered into the solid angle  $d\Omega$  is also proportional to the differential cross section, therefore

$$dN \propto \frac{d\sigma}{d\Omega} d\Omega = \frac{d\sigma}{d\Omega} \sin \theta d\theta \times d\phi \quad (\text{C-6})$$

Averaging over  $\phi$ . As it is often the case both the incident beam and the atoms in the target are unpolarized (you have to work pretty hard to polarize the beam or the target, if at all possible) and for this reason the distribution of the scattered particles is cylindrically symmetric around the beam axis, or around the  $z$  axis in our case. This means that the cross section in Eq. C-6 is independent of  $\phi$  and we can then readily integrate this equation over  $\phi$ . The region of integration is shown in Fig. C-4.

$$dN(\theta) = \int_0^{2\pi} \sigma(\theta) \sin \theta d\theta d\phi = \sigma(\theta) 2\pi \sin \theta d\theta \quad (\text{C-7})$$

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The *Klein-Nishina* cross section given in Eqs. 30 and 32, and in General Appendix D, is the result of averaging over the two photon polarizations and the two electron polarizations, for this reason it does not depend on  $\phi$ .

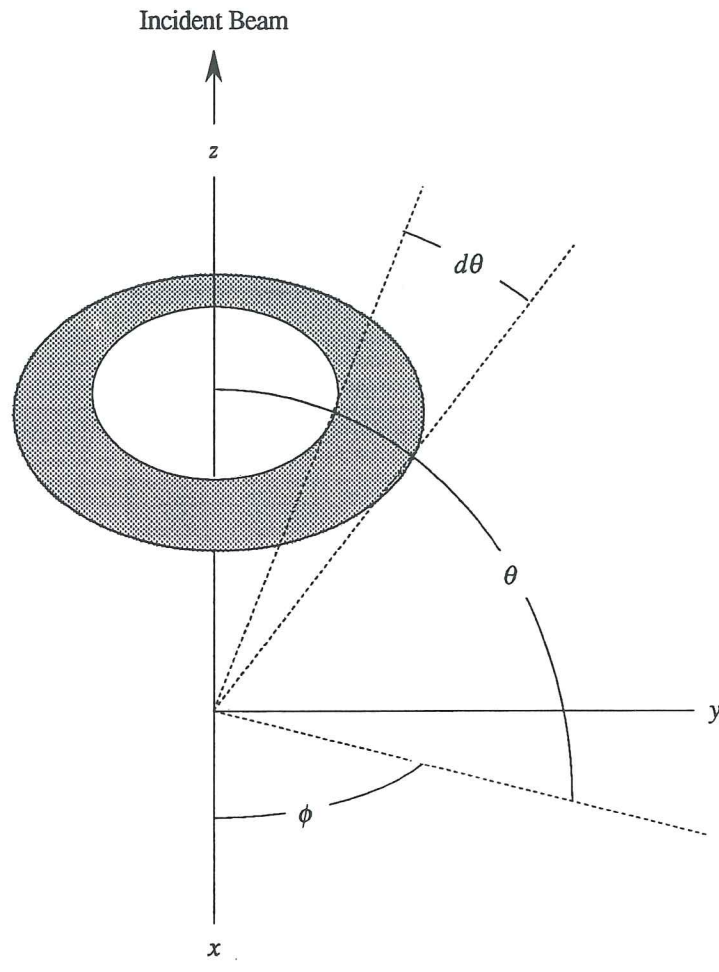


Figure C-4