APPENDIX B

CROSS SECTIONS

1- Differential cross sections

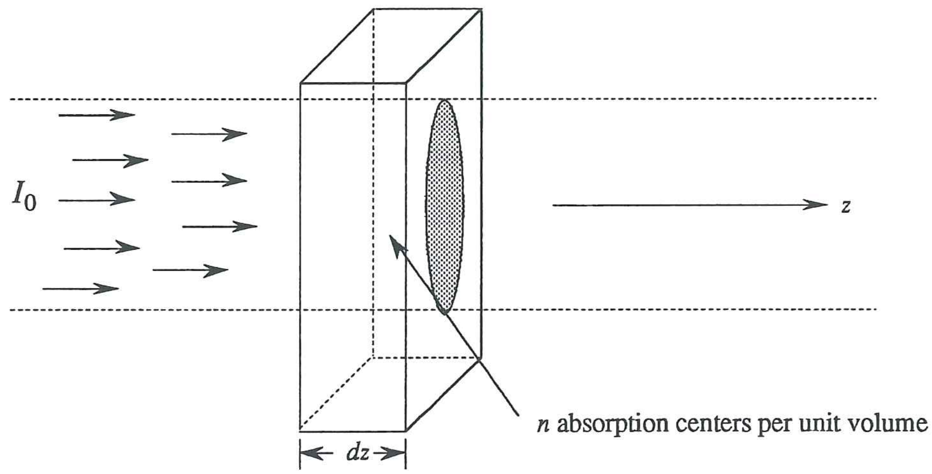

Figure B-1

Consider a target whose cross sectional area is larger than the beam cross section, as shown in Fig. B-1. I_0 is the incident number of beam particles per second, n the scattering centers per unit volume, and dz the target thickness. $d\eta$ is the number of "η" particles (not necessarily the same type as the incident particles) emitted into the solid angle $d\Omega$ per second, see Fig. B-2 (for the definition and details of solid angle see General Appendix C). The definition of the differential angular cross section $\dfrac{d\sigma}{d\Omega}$ is given by

$$\frac{d\eta}{d\Omega} = \left(n\,dz \frac{d\sigma}{d\Omega} \right) I_0 \tag{B-1}$$

In the same manner we can define the differential energy cross section $\dfrac{d\sigma}{dE}$. If the number of "η" particles emitted in all directions within the energy range E to $E + dE$ is $d\eta$, the differential energy cross section is defined by

$$\frac{d\eta}{dE} = \left(n\,dz \frac{d\sigma}{dE} \right) I_0 \tag{B-2}$$

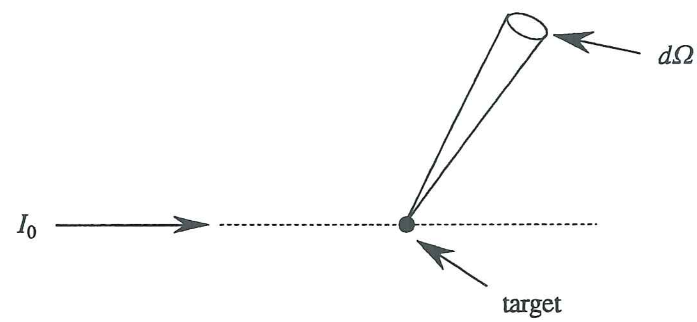

Figure B-2
I_0 particles area incident on the target.

We can calculate the total cross section for a particular process by integrating the differential cross section over the relevant variable,

$$\sigma_T = \int \frac{d\sigma}{d\Omega} d\Omega \qquad \text{(B-3)}$$

or

$$\sigma_T = \int \frac{d\sigma}{dE} dE \qquad \text{(B-4)}$$

With these definitions the total number of "η" particles emitted per second is given by

$$\eta = (n\,dz\,\sigma_T) I_0 \qquad \text{(B-5)}$$

Eqs. B-1, B-2 and B-5 are valid if the attenuation of the incident beam is negligible, otherwise you must take account of the incident beam attenuation. See the following section.

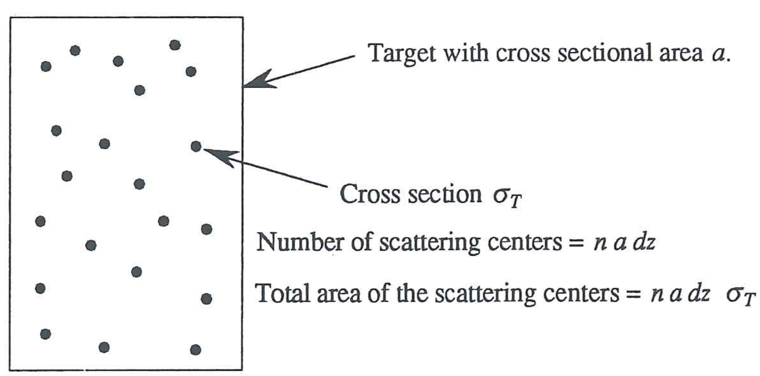

Figure B-3
The target as viewed by the incident beam.

Geometrical Interpretation. Notice that the dimension of σ_T is an area (also $d\sigma$). Looking at the target from the point of view of the incident beam we see the picture shown in Fig. B-3. The area obscured by the scattering centers is

$$\text{area obscured} = n\,a\,dz\,\sigma_T \qquad \text{(B-6)}$$

therefore the fraction of the target area obscured by the scattering centers is

$$fraction = \frac{n\,a\,dz\,\sigma_T}{a} = n\,dz\,\sigma_T \qquad \text{(B-7)}$$

and therefore the fraction of the interactions yielding "η" particles is

$$fraction = \frac{\eta}{I_0} = n\,dz\,\sigma_T \qquad \text{(B-8)}$$

which is identical to Eq. B-5.

2- Total absorption cross section.

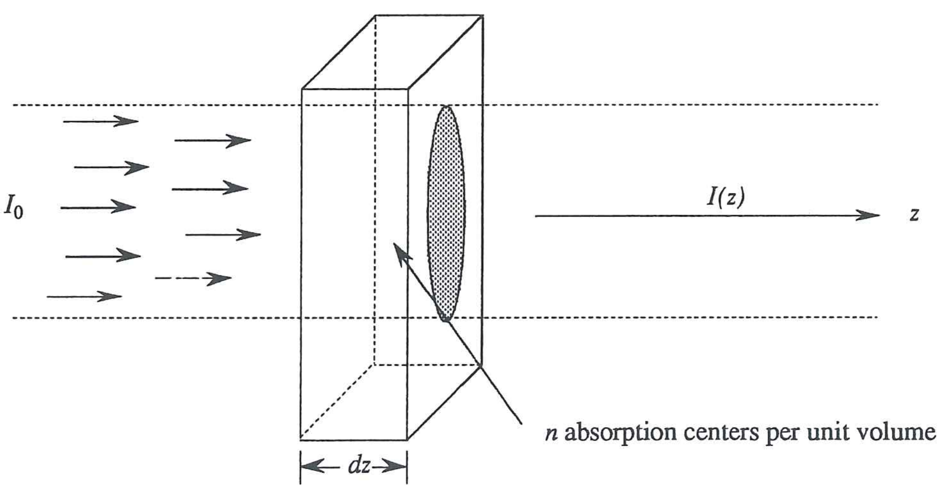

Figure B-4

Again, consider a target whose cross sectional area is larger than the beam cross section, as shown in Fig. B-4. $I(z)$ is the flux of beam particles at the depth z inside the target, and the total cross section per one absorber center is σ_{tot}. There is a very importat difference between σ_T defined in the previous section and σ_{tot} used in this section. σ_T is the total cross section for the production (or scattering) of "η" particles, while σ_{tot} is the cross section for all interactions between the beam particles and the target particles. The decrease in the flux of the incident beam in the length dz in the target is given by

$$dI(z) = -n\, dz\, I(z)\ \sigma_{tot} \qquad \text{(B-9)}$$

where $d\,I(z)$ is the number of beam particles absorbed by the target. From Eq. B-9

$$\frac{dI(z)}{I(z)} = -n\,\sigma_{tot}\, dz \qquad \text{(B-10)}$$

If the target is not very thin we have to integrate Eq. B-10 to obtain the number of beam particles left in the beam,

$$\ln\frac{I(z)}{I_0} = -n\,\sigma_{tot}\ z \qquad \text{(B-11)}$$

or

$$I(z) = I_0 e^{-n\sigma_{tot} z} \qquad \text{(B-12)}$$

Trivial remark. As noted earlier both the differential cross section and the total cross section have the dimensions of an area. For a classical collision between two billiard balls of radius R_1 and R_2, the balls collide if the distance of separation of their centers is less than $R_1 + R_2$. The total cross-section for this case is just the area $\pi\,(R_1 + R_2)^2$ as shown in Fig. B-5.

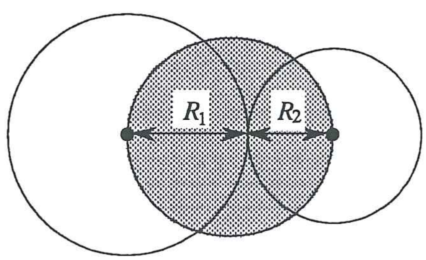

Figure B-5

The total cross section for two hard billiard balls.

3- Mass attenuation coefficients. These are defined by the coefficient of z in Eq. B-12,

$$\mu = n\,\sigma_{tot} \qquad \text{(B-13)}$$

If the absorption centers are atoms, n is given by

$$n = \frac{\rho N_A}{A} \qquad \text{(B-14)}$$

Where ρ is the target's density, A the target's atomic mass, and N_A Avogadro's number. Inserting Eq. B-14 into Eq. B-13 we get,

$$\mu = \frac{\rho N_A}{A} \sigma_{tot} \quad (\mathrm{cm}^{-1}) \qquad \text{(B-15)}$$

Often the attenuation coefficients are expressed as the ratio of μ to ρ (why?),

$$\frac{\mu}{\rho} = \frac{N_A}{A} \sigma_{tot} \quad (\mathrm{cm}^2/g) \qquad \text{(B-16)}$$

If the absorption centers are the atomic electrons then you must multiply Eqs. B-14 through B-16 by Z, the atomic number.

For further details see *Leo* 2.1.1, 2.1.2 and 2.1.3