Experiment 34

LIFETIME OF A NUCLEAR EXCITED STATE

INTRODUCTION

In this experiment we will measure the time interval between detections of two different gamma rays to measure the lifetime of a nuclear excited state. You will measure the 14.4KeV state of ^{57}Fe, the daughter nucleus of a ^{57}Co decay (Figure 1). By measuring the time between the emission of a 122KeV photon and a subsequent 14.4KeV photon emitted by the same nucleus we can collect data to determine the lifetime of this excited nuclear state, which is approximately 100 nanoseconds.
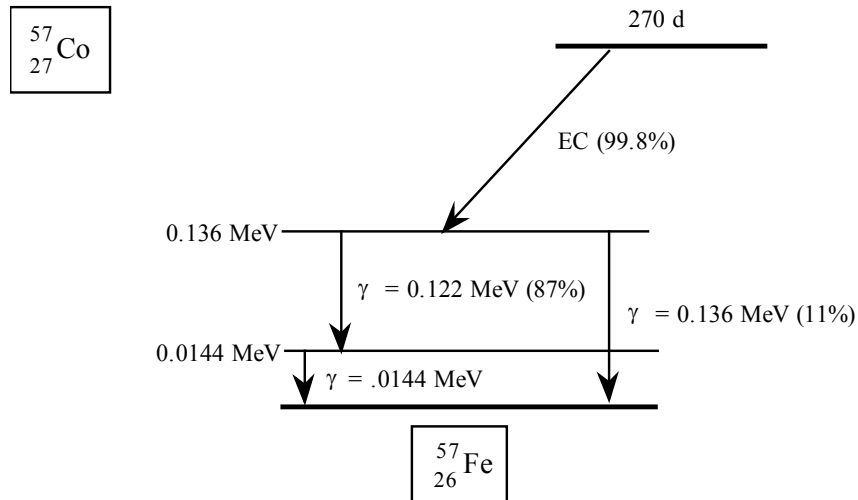

Figure 1: Decay of ^{57}Co to ^{57}Fe. The experiment will measure the lifetime of the 14.4KeV excited state of the ^{57}Fe daughter nucleus. 14.4KeV gamma photons are rarely emitted when this state decays; it usually loses its energy through internal conversion.
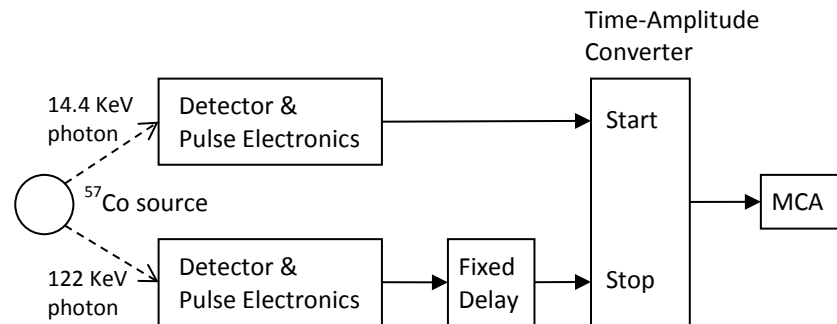
EXPERIMENT SETUP

Figure 2: General schematic of the measurement setup. 14.4KeV and 122KeV photons are detected and processed along independent, parallel paths. The outputs of these paths are sent to a Time-Amplitude Converter (TAC), which measures the time separation between the detections and sends the result to the MCA to be cataloged and displayed. A fixed delay of approximately 1 microsecond is inserted in the 122KeV photon signal path so that it arrives at the TAC after the 14.4KeV pulse.

The set of equipment used to perform the lifetime measurement appears complicated at first glance, but it is actually quite straightforward. Two photon detection channels are required, one each for the 122KeV and 14.4KeV photons (Figure 2). Each channel uses a NaI scintillator (and electronics) to detect its photon (as in Experiment 30a). Figure 3 shows a block diagram of the electronics modules used in each detection channel.
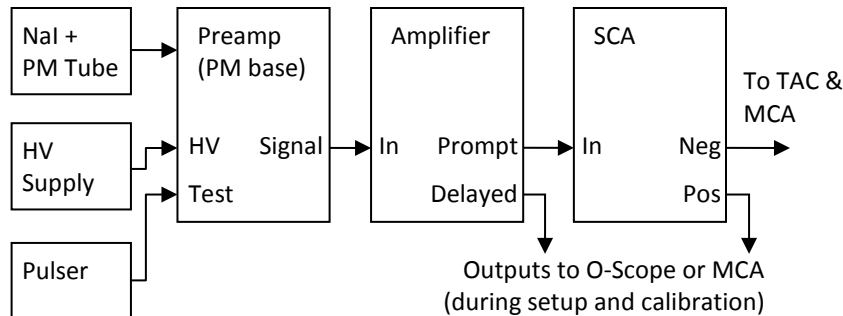

Figure 3: Detector and pulse electronics for each of the two photon detection paths. The detector is a NaI scintillator attached to a photomultiplier (PM) tube. The detector preamp is built into the PM tube base. The pulser is used to calibrate the TAC. The Single Channel Analyzer (SCA) outputs a digital pulse whenever an event of the correct energy is detected. Its output pulse can be delayed by up to 1 microsecond. This delay is used in the 122KeV path (see Figure 2).

Time-Amplitude Converter (TAC)

The key electronics module used to perform the measurement is a Time-Amplitude Converter (TAC). This device measures the time interval between pulses presented to its start and stop inputs. The time interval is converted to a voltage by a precision circuit. This voltage is output as a pulse from the TAC. By sending this voltage to a standard Multichannel Analyzer (MCA), as was used in Experiment 30a, the experiment can generate a histogram of the time intervals observed between pairs of start and stop pulses. The TAC has an overall linearity of better than 0.1% of full scale but must still be calibrated, along with the MCA, for accurate results.

Single Channel Analyzer (SCA)

A Single Channel Analyzer (SCA) acts like a single channel of a MCA spectrometer; it produces a digital output pulse when its input falls between adjustable upper and lower voltage levels (called *discriminators*). The upper and lower levels are set so that the channel output is generated only for input gamma photons of the proper energy.

The SCA has two outputs: a fast gate which is sent to the TAC for timing and a slower square pulse which will be useful when setting the upper and lower levels. The SCA also has an adjustable delay of up to 1uS before it generates an output pulse. This delay is used in the 122KeV detection path (shown as the "fixed delay" block in Figure 2).

Calibrated Delay Unit

In order to calibrate the time axis of our spectra there are 4 accurate delay modules. These units contain accurate lengths of coiled cable which can be switched in and out as desired. The units are inserted between the 14.4KeV detector SCA and the TAC input. They are used during the time calibration segment of the experiment.

Pulser

The pulser is used to generate a constant stream of test pulses when calibrating the time axis. When activated its pulses are introduced simultaneously into both photon detection paths (via a test input on each detector preamp).

UCS MCA Spectrometer

You'll use either a UCS-20 or a UCS-30 Multichannel Analyzer (MCA) both for taking time spectra from the TAC output pulses and for taking scintillator energy spectra when setting up the experiment. As with the MCAs used in experiment 30a it produces a histogram of the peak voltages of its signal input pulses.

The UCS MCA also has a *gate input*. When the gate input is left disconnected, every input signal pulse is analyzed by the MCA; with the gate input connected only those signal pulses present at the same time as a pulse on the gate input are analyzed. This feature allows the SCA levels to be set by connecting the gate to the SCA output while the MCA signal input is connected to a (delayed) amplifier output. In this configuration the MCA displays an energy spectrum of the pulses which fall within the limits set by the SCA lower and upper level discriminator settings so that you can accurately set these levels to limit the detections to the desired 122KeV and 14.4KeV photon events.

EXPERIMENT PROCEDURE

The procedure may be divided into four phases:

1. Initial setup

2. Time calibration

3. 122KeV and 14.4KeV peak identification and SCA setup

4. Decay time data collection

Each of these phases will be discussed in detail below.

Initial Setup

Check that each module is set up as follows:

TC 812 Pulser	
☐ Pulse Height:	5.00
☐ Relay:	Off
☐ Pol.:	–
☐ Output:	5 V, 0.4 mS

TC 861A Time-Amplitude Converter (TAC)	
☐ F.S. Time:	2 μS
☐ Gate:	Coincidence
☐ Strobe:	Internal

425A Delay
☐ All switches to **Delay Out** (left)

553 Timing SCA		
☐ Upper Level:	full clockwise (10 V)	
☐ Lower Level:	0.40 (not 4.00!)	
☐ Delay (μS):	14.4 KeV module: 0.1	122 KeV Module: 1.1
☐ Mode:	Nor (normal)	

435A Active Filter amplifier

☐ Coarse Gain: 16

☐ Gain switch: High

☐ Fine Gain: 7.00

☐ Mode switches: Pos, Bipolar

5kV High Voltage Supply (both channels A and B)

☐ Meter range: 1 mA

☐ HV power: Off

☐ Trip mode: Auto Reset

☐ HV dials: 2.00 (will set voltage to 1000V)

UCS Universal Computer Spectrometer (MCA)

☐ Power switch: On

Software (*UCS20* or *USX USB*):

Settings: High Voltage/Amp/ADC:
☐ Conversion Gain: 256
☐ Voltage: Off
☐ Discriminators: LL 5; UL 256

Settings: ROI: Clear All ROIs

Settings: Mode (*USX USB* software only)

☐ PHA – Direct In

Time Calibration

☐ First ensure that:
   (1) the MCA SIGNAL INPUT cable is connected to the TC 861A TAC output;
   (2) the MCA GATE INPUT cable is disconnected;
   (3) all modules are configured with their settings as described above; and
   (4) the TC 812 pulser output **A** is connected to a ×10 attenuator, whose output is connected to both preamplifier Test inputs on the scintillation detectors.

☐ Turn the TC 812 pulser RELAY switch to ON. Use the oscilloscope channel 1 input cable to check each of the DELAYED outputs of the two 435A amplifiers. Each output should show positive pulses of about 4–8V amplitude and 5μS duration. If not, adjust the TC 812 PULSE HEIGHT control until the pulse amplitude is within this range.

☐ Now use the oscilloscope channel 1 input to check each of the POS outputs of the 553 SCA's. Each should show +5V to +6V square pulses of about 500nS duration. If an SCA doesn't show pulses, check that its Lower Level is set below 1.00, and that its Upper Level knob is fully clockwise. If the SCA Lower Level is set too low, then electronics noise will generate additional SCA output pulses which will corrupt the calibration data.

☐ Acquire a spectrum using the UCS software. You should see a strong, narrow peak at about 1/2 of full scale (UCS-30) or 2/3 of full scale (UCS-20). This peak corresponds to a simultaneous detection in both the 14.4KeV and 122KeV channels; the relative pulse arrivals at the TAC correspond to a delay of approximately 1μS in the 122KeV channel, as set by the DELAY knobs on the two 553 SCA's. Turn the pulser RELAY switch OFF if you need to stop the pulses, but simply stopping and then resuming the MCA acquisition while adjusting the channel delays works quite well.

☐ Insert various amounts of delay to the 14.4KeV signal using the switches on the four 425A delay modules. Delays should be added in increments of at least 25nS to avoid excessive overlap between adjacent peaks in the spectrum. A total of about 250nS may be added, which should move the original peak to the left by approximately 35–50 channels. Estimate the conversion slope (nS/channel) from the spectrum.

☐ Return the delay module switches all to their DELAY OUT positions and turn the 122KeV SCA delay knob to its center position (~0.6μS). Now repeat the above step. This will extend your time calibration into a different segment of the MCA spectrum.

☐ Make sure you save your calibration spectra in .tsv format.

☐ Turn the TC 812 pulser RELAY to OFF. Reset all 425A delay switches to DELAY OUT (left).

☐ Reset the 122KeV 553 SCA DELAY knob to its fully clockwise position (1.1µS), and check that the 14.4KeV SCA DELAY knob is fully counterclockwise (0.1µS). This completes the time calibration.

122KeV and 14.4KeV peak identification and SCA setup

For this part of the procedure, you must examine the energy spectra of the two scintillation detector outputs and identify the 122KeV and 14.4KeV gamma ray photons in these spectra. You will then set the upper and lower levels of the two 553 SCAs so that they respond to detections of their corresponding photon events.

122KeV channel:

☐ Turn the HV switches on the high voltage supply to ON. Both HV LEDs should illuminate. Check each of the high voltages by connecting a digital multimeter to its MONITOR output. Each voltage should be 1000V ± 50V. If a voltage is above 1100V, immediately turn its HV switch to OFF and seek assistance from your TA.

☐ Insert a ^{57}Co source between the two scintillators. Use tweezers to support the source. The source front surface should be facing the 14.4KeV scintillator and the source should be up against the centers of the front surfaces of the two scintillators.
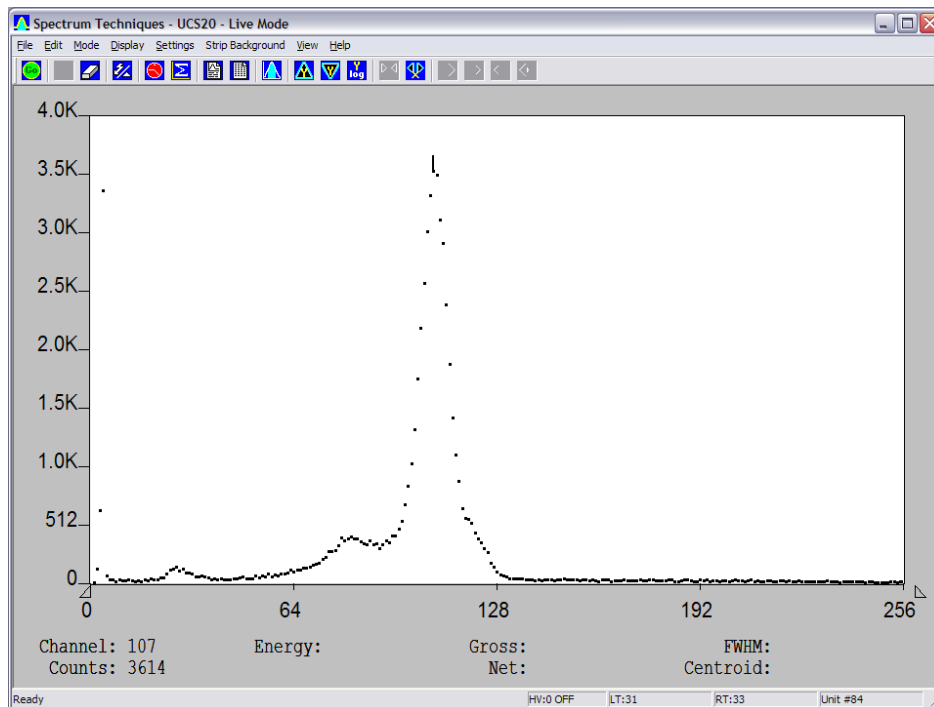

Figure 4: A typical UCS-20 spectrum display of the 122KeV detector. The cursor is positioned on the large full-energy peak of the 122KeV gamma photon. Also visible are the much smaller Compton edge, backscatter peak, and 136KeV full-energy peak. The MCA signal input was connected to the 435A amplifier DELAYED output for this spectrum.

☐ Connect the MCA SIGNAL INPUT cable to the 122KeV 435A Active Filter amplifier DELAYED output. The MCA GATE INPUT cable should be disconnected.

☐ Take a MCA spectrum; the result should be similar to Figure 4. Identify the 122KeV peak and the 83KeV backscatter feature. Save the spectrum or an image of it.

☐ Define a Region of Interest (ROI) which includes just the 122KeV peak.

☐ Connect the MCA GATE INPUT cable to the 122KeV 553 Timing SCA POS OUT.

☐ Start another spectrum and adjust the 553 SCA UPPER LEVEL and LOWER LEVEL controls until only the ROI which includes the 122KeV peak is included in the spectrum (keep erasing and restarting the spectrum to make sure it includes only the ROI). Save a spectrum (or its image) which shows how you have set the SCA levels to include the peak. If the count rate is too high, the levels may be hard to set. Pull the detector a little bit away from the source to lower the count rate while you set them.

☐ Disconnect the MCA SIGNAL INPUT and MCA GATE INPUT cables from the amplifier and SCA outputs and clear the ROI you defined.
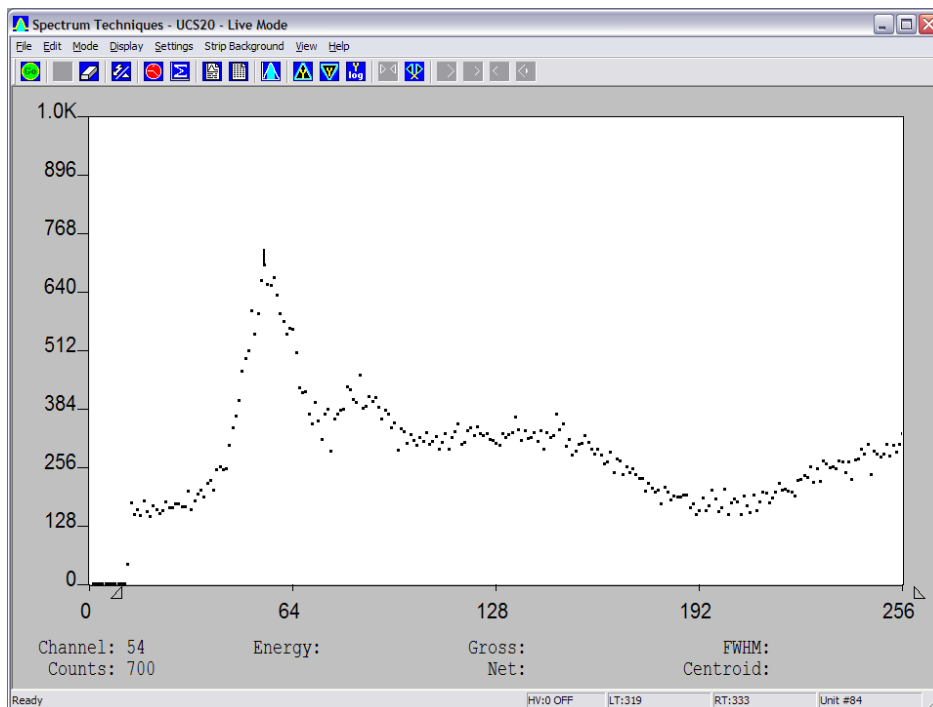

Figure 5: A fairly well-defined UCS-20 spectrum display of the 14.4KeV detector. The cursor is positioned on the weak full-energy peak of the 14.4KeV gamma photon. Also visible are an X-ray peak at approximately 20KeV and the Compton spectrum of the 122KeV gamma. The MCA signal input was connected to the 435A amplifier DELAYED output for this spectrum. The relative heights of the 14.4KeV and 20KeV peaks may be different depending on the particular [57]Co source you are using, and your source may be weaker, building the spectrum rather slowly.

14.4KeV channel:

☐ Connect the MCA SIGNAL INPUT cable to the 14.4KeV 435A Active Filter amplifier DELAYED output. The MCA GATE INPUT cable should be disconnected.

☐ Take a MCA spectrum; the result should be similar to Figure 5. Identify the 14.4KeV peak and the X-ray fluorescence peak at about 20 KeV. Save the spectrum or an image of it.

☐ Define a Region of Interest (ROI) which includes just the 14.4KeV peak.

☐ Connect the MCA GATE INPUT cable to the 14.4KeV 553 Timing SCA POS OUT.

☐ Start another spectrum and adjust the 553 SCA UPPER LEVEL and LOWER LEVEL controls until only the ROI which includes the 14.4KeV peak is included in the spectrum (keep erasing and restarting the spectrum to make sure it includes only the ROI). Save a spectrum (or its image) which shows how you have set the SCA levels to include the peak.

☐ Disconnect the MCA SIGNAL INPUT and MCA GATE INPUT cables from the amplifier and SCA outputs and clear the ROI you defined.

This completes the SCA setup.

Decay time data collection

Now you are ready to collect the decay lifetime data. The basic idea is to collect the time separations between pairs of 122KeV and 14.4KeV photon events. The distribution of the delay times should be described by an exponential decay.

Clearly, the experiment could proceed by starting the time measurement with the detection of a 122KeV photon. The timing would then be stopped when a 14.4KeV event is detected. If another 122KeV photon is detected before seeing a 14.4KeV, then reset the timer and start timing again (assume that the 14.4KeV photon was missed).

Note, however, that the 122KeV detection rate is much higher than the rate of 14.4KeV detections (recall that the 14.4KeV excited state decays via a conversion electron much more frequently than a gamma emission, and also note that many 14.4KeV photons may be absorbed by the aluminum cover of the detector before reaching the scintillator).

Under these circumstances it is a more efficient procedure to use a 14.4KeV detection to trigger the decay timing, and use the 122KeV detection to stop the timing (as pointed out by Chris Wegg). This method results in a noticeably lower false detection background and provides better data statistics. But how can we start timing with the detection of a 14.4KeV photon when the 122KeV photon (which signaled the population of a 14.4KeV excited state) has already been emitted?
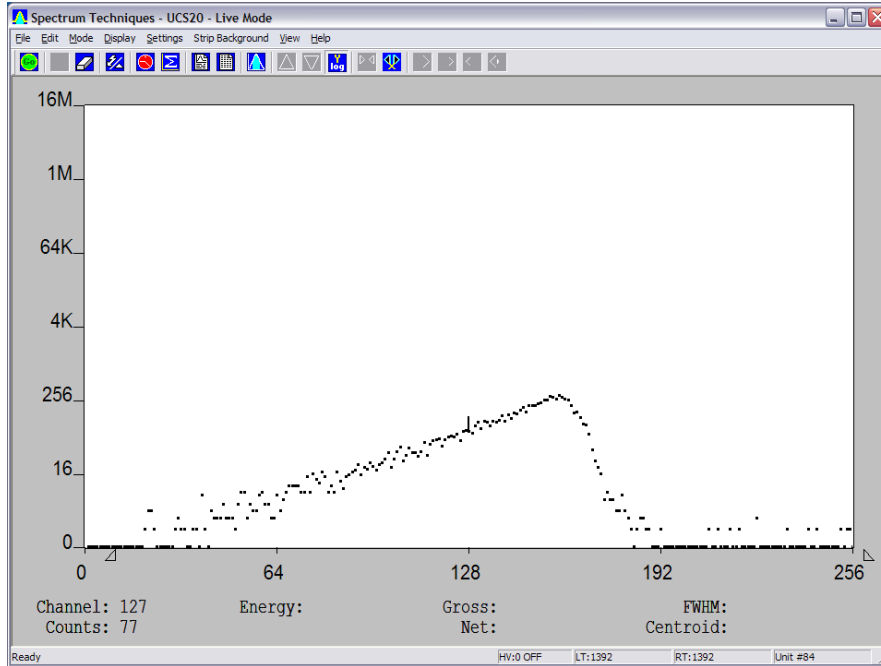
Figure 6: UCS-20 spectrum display of the Time-Amplitude Converter (TAC) output showing decay lifetime data. Timing is started with detection of a 14.4KeV photon and is stopped by a 122KeV photon detection. The 122KeV detection is passed through a fixed delay of approximately 7 mean lifetimes so that it arrives at the TAC after the 14.4KeV detection. Because the 14.4KeV starts the timing, longer delays correspond to lower channel numbers (see equation (1)). The peak of the spectrum corresponds to a very short time interval between the 122KeV and 14.4KeV events.

The solution is to delay the channel with the 122KeV detections by many decay lifetimes, so that its detection is still available to stop the timing once the 14.4KeV detection starts it. As the observed time separation between the 14.4KeV detection and the delayed 122KeV detection grows shorter, the longer the time between the actual 122KeV and 14.4KeV emissions becomes. This is the method you will use.

If the fixed time delay of the 122KeV detection is T_{delay} and the time interval between the 122KeV and 14.4KeV emissions is t, then the observed time interval (starting on the 14.4KeV detection and stopping on the delayed 122KeV detection) is given by:

$$t_{obs} = T_{delay} - t \tag{1}$$

☐ Connect the MCA SIGNAL INPUT cable to the TC 861A TAC output, TAC OUT. *Make sure the* MCA GATE INPUT *cable is disconnected*.

☐ Make sure that all delays on the 425A Delay modules have been deselected (all switches to DELAY OUT).

☐ Make sure that the 553 SCA DELAY controls are set as follows:

14.4KeV:	Fully counterclockwise (0.1μS)
122KeV:	Fully clockwise (1.1μS)

☐ Start a new MCA spectrum. Use the LogY scale setting and make sure that you see at least some events occurring every now and then near channels 100–200. If everything is working as it should, you will start to build up the decay lifetime spectrum. You will need to collect data for the rest of the lab time (at least 1 hour).

☐ Stop and save the spectrum every 20 minutes or so. Immediately resume the spectrum once you have saved it. Figure 6 shows a typical spectrum after about 20 minutes of data collection.

Securing the experiment

Once you have completed data collection:

☐ Stop the UCS spectrum acquisition.

☐ High Voltage Supply: turn both HV switches to OFF.

☐ Return the ^{57}Co source to its lead container.

ANALYSIS

Time calibration

The spectra you collected using the pulser provide the data you need to evaluate the linearity of the channel–time relationship and the conversion factor $(time / channel)$. Determine this conversion factor from your calibration data.

Decay analysis

Use *CurveFit* to analyze your decay timing data (Figure 6). The count data (Y values) should obey Poisson statistics; use this fact to set the uncertainties in the Y values.

The channel events well to the right of the maximum (above channel 192 in Figure 6) show the nearly constant false detection (background) rate. Fit a constant function to this data segment to determine the mean number of background counts per channel.

The data should represent a decaying exponential in time. Because the start triggering was done using the 14.4KeV detections, however, the raw channel data show an exponential rise. You must therefore transform the X (time) values before fitting the data. Equation (1) provides the information you need to perform the appropriate transformation to the X data values. Use the *CurveFit* palette menu selection:

<div align="center">Modify data points→Algebraic Transforms→User-Defined</div>

For the best fit results the transformed decay data should start near X = 0. You should correct the Y count data as well by subtracting the mean background counts/channel.

Once you have transformed the X and Y values to show a decaying exponential (with constant background rate also subtracted) you then need to select a subset of data points which include only an exponentially decaying section of the spectrum. Note that if you include points too close to the initial peak in the decay you will introduce a systematic error in the fit because of the finite time resolution of the experiment and because there are often an excess number of events with ~0 delay (cause by a single photon Compton scattering between the two detectors). Another source of systematic error in the fit may be introduced if you include too many channels with very few counts, because the Poisson distribution of counts is very different from a Gaussian approximation in this case (this problem might be mitigated by rebinning the data to aggregate the counts in a few adjacent channels. See the *CurveFit* palette menu selection: Modify data points→Transform Gamma Spectrum→Rebin Data).

By fitting an exponential to the decay data you may estimate (with an uncertainty) the half-life of the ^{57}Co nucleus 14.4KeV excited state. Refer to Prelab Problem 2 for the formula to calculate the half-life of an exponential decay. The published half-life is approximately 98nS.

PRELAB PROBLEMS

1. Identify the following features in the ^{57}Co scintillator energy spectrum shown in Figure 7 below. The radioactive source is embedded in a Rhodium foil.

   a) 122KeV full energy peak.

   b) 136KeV full energy peak.

   c) 14.4KeV full energy peak.

   d) Backscatter peak of the 122KeV photon.

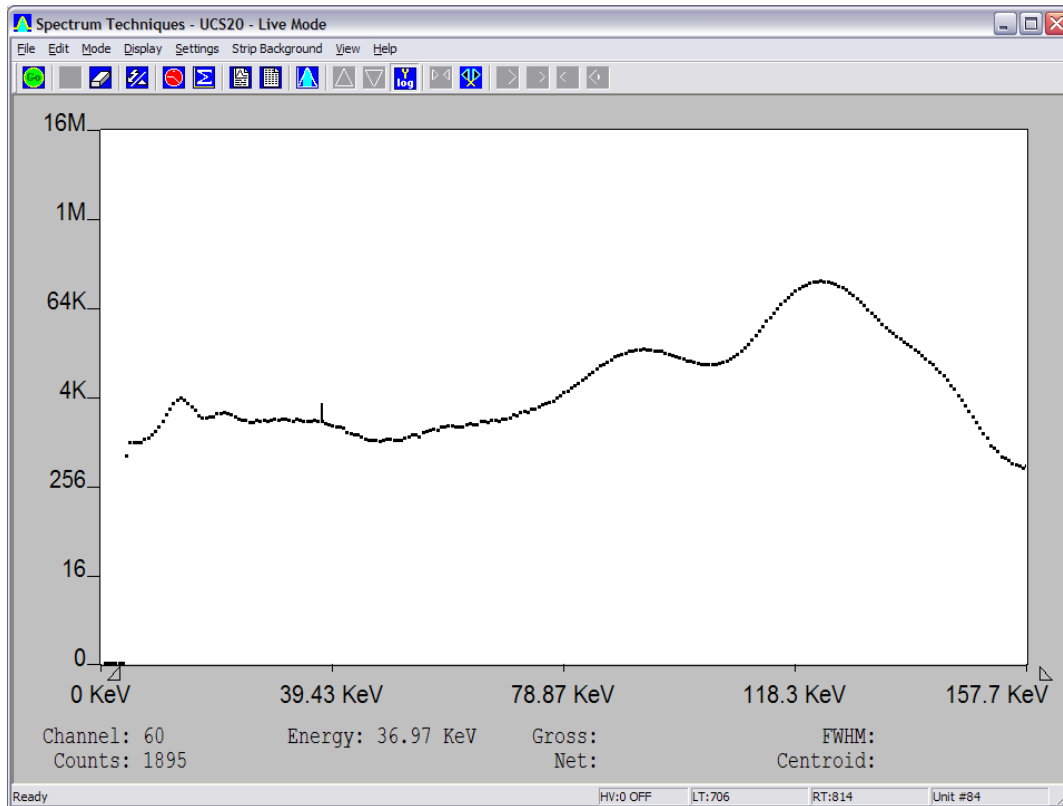   e) Rhodium X-ray fluorescence due to photoelectric absorption of 122KeV photons.

Figure 7: Cobalt-57 photon energy spectrum obtained with a scintillation detector.

2. A nuclear excited state has a constant probability of decay in any short time interval dt which is independent of how long the nucleus has been excited, i.e.: $dP_{decay} = (1/\tau) dt$ for some constant characteristic time τ. Show that:

   a) If the nucleus entered the excited state at time $t = 0$, then the probability that it is still in the excited state at any subsequent time is: $P_{excited}(t) = e^{-t/\tau}$.

b) If the nucleus entered the excited state at time $t = 0$, then the probability that it remains excited until time t but then decays between times t and $t + dt$ also decreases exponentially as $e^{-t/\tau}$.

c) The time τ is called the *mean lifetime*. How is the *half-life* ($\tau_{1/2}$), the time required for the probability to reach ½ (i.e.: $P_{excited}(\tau_{1/2}) = 1/2$), related to τ?

3. Assume that the excited state of a quantum system has a mean energy of $E_0 = \hbar\omega_0$ (the ground state is assumed to have energy 0) and mean lifetime τ. Then the time variation of the wave function of the excited state can be written as:

$$\psi(t) = \psi(0) e^{-i\omega_0 t} e^{-t/2\tau} \,, \quad t \geq 0$$
$$\psi(t) = 0 \qquad\qquad\qquad , \quad t < 0$$

where $\psi(0)$ is the normalized wave function of the state (as a function of position) at time $t = 0$ (the factor of 2 arises because τ characterizes the probability, which varies as $|\psi(t)|^2$).

The Fourier transform of this function of time gives the wave-function as a function of angular frequency $\psi(t) \to \Psi(\omega)$. The Fourier transform is defined as:

$$\Psi(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(t) \, e^{i\omega t} \, dt$$

Show that $\Psi(\omega)$ is a Lorentzian, with full width at half-maximum $\gamma = 1/\tau$:

$$\Psi(\omega) = \frac{1}{\sqrt{2\pi}} \times \frac{1}{i(\omega_0 - \omega) + \frac{\gamma}{2}} \psi(0)$$

$$|\Psi(\omega)|^2 = \frac{1}{2\pi} \times \frac{1}{(\omega_0 - \omega)^2 + \left(\frac{\gamma}{2}\right)^2} |\psi(0)|^2$$

If we define $\Delta E = \hbar\gamma$, $\Delta t = \tau$, then the *Uncertainty Principle* relation between them for the case of an excited state with a finite lifetime is: $\Delta E \, \Delta t = \hbar$.