

Experiment 25

BALMER LINES OF HYDROGEN AND DEUTERIUM
and the proton-electron mass ratio

Introduction	1
Theory	1
The Monochromator	2
Prelab Problems	7
Procedure	9
Data Analysis	13

INTRODUCTION

In this experiment you will use the relative positions of the emission lines of hydrogen and deuterium to determine the proton/electron mass ratio. Hydrogen, the simplest of atomic systems, has an emission spectrum which is consistent with an elementary theory derived from the quantum *Uncertainty Principle* with a minimum of additional assumptions, as long as you don't examine the spectrum at very high resolutions. The energy levels of the electron-nucleus system depend on its reduced mass, which is different for hydrogen and deuterium. Consequently, the emitted wavelengths of the hydrogen and deuterium spectra slightly differ, and careful measurements of the wavelength differences provide for a determination of the ratio of the reduced masses of the two atoms. To collect your data you will use a research-grade spectrographic instrument called a *monochromator*, because it transmits only a narrow band (~ 0.1 nm) whose center wavelength can be tuned through the visible spectrum.

THEORY

A simple theory of the hydrogen atom energy levels is presented in the notes for Experiment 25a (a Physics 6 experiment). The energy levels are:

$$\begin{aligned} E_n &= \frac{1}{n^2} E_{\text{ground}}; \quad n \in \{1, 2, 3, \dots\} \\ -E_{\text{ground}} &= \frac{\mu e^4}{2\hbar^2} = \frac{1}{2} \alpha^2 \mu c^2 \end{aligned} \quad (1)$$

where we use Gaussian units, and μ is the *reduced mass* of the electron-nucleus system. The final expression for E_{ground} is in terms of the rest-energy μc^2 and the dimensionless *Fine Structure Constant*, $\alpha \equiv e^2/\hbar c \approx 1/137$.

Because the nucleus has a finite mass, motions of the electron and the nucleus are about their center of mass; as you will learn in classical mechanics lecture, this two-body motion may be analyzed by considering the equivalent *central force problem* of a single body in motion about a fixed force center. In order for the two systems to be equivalent, the mass of the single body is given by the *reduced mass* of the electron and nucleus:

$$\mu = \frac{m_e m_N}{m_e + m_N} = m_e \left(\frac{1}{1 + m_e/m_N} \right) \quad (2)$$

In the limit that the nuclear mass m_N becomes infinite, the reduced mass is just the electron mass, m_e . Because the nuclear mass is finite, however, μ is slightly less than the electron mass.

A hydrogen nucleus consists of a single proton, whereas a deuterium nucleus is a bound proton-neutron pair. Chemists can perform careful experiments to determine the mass ratio of a hydrogen and deuterium nucleus: $m_p/m_D = 0.5003$. The ratio of the reduced masses of the two atoms is thus (assuming that $m_p/m_e \gg 1$):

$$\frac{\mu_H}{\mu_D} = \frac{1 + m_e/m_D}{1 + m_e/m_p} \approx 1 + \frac{m_e}{m_D} - \frac{m_e}{m_p} = 1 - \frac{m_e}{m_p} \left(1 - \frac{m_p}{m_D}\right) \approx 1 - 0.500 \frac{m_e}{m_p} \quad (3)$$

From equation (1), this reduced mass ratio is just the ratio of the energies of any pair of corresponding energy levels in the two atoms.

THE MONOCHROMATOR

The monochromator is a Spectral Products model DK-480. It uses a diffraction grating ruled onto the surface of a flat mirror to disperse light from the source. A schematic diagram of the optical arrangement of the DK-480 is shown in figure 1, which is known as a *Czerny-Turner* arrangement.

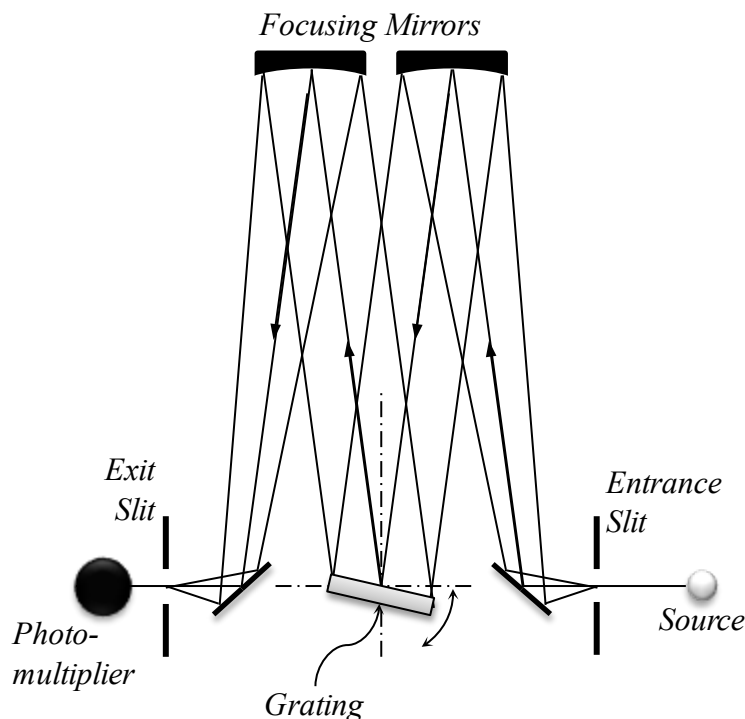


Figure 1: The optical arrangement of the DK-480 monochromator, a *Czerny-Turner* design. All optical elements are at a fixed position except for the grating, which is rotated to select the wavelength of the source which reaches the exit slit. The entrance and exit slits have adjustable widths to control the wavelength resolution. The focusing mirrors each have a focal length of 0.48 meter. The grating has 1200 grooves/mm and is approximately 6.8 cm square.

How does the angle of the grating control the output wavelength of the monochromator? To answer this question we must consider the interference of the waves scattered by the grooves on the grating. Consider the close-up diagram of the waves and the grooves shown in figure 2.

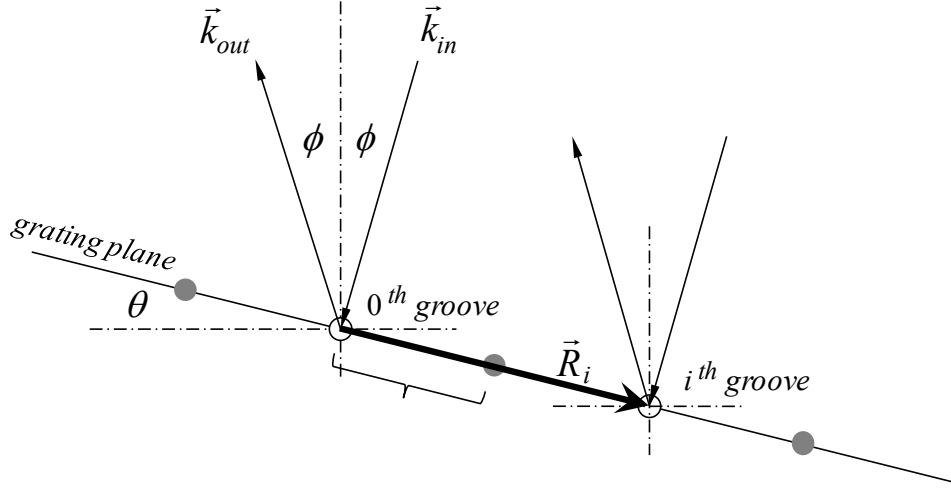


Figure 2: The plane wave from the source (wave vector \vec{k}_{in}) shines on the grating with grooves spaced by distance d (the grooves are oriented perpendicular to the plane of the figure). Each groove scatters the incoming wave, emitting a cylindrical wavefront. We are interested in that part of each scattered wave with wave vector \vec{k}_{out} , which is in the direction of the output focusing mirror. The grating and the wave vectors are oriented with the angles θ and ϕ , as shown.

The first focusing mirror takes the image of the input slit and projects it to a virtual image at infinity, so that the light from the source is a plane wave as it illuminates the grating. If the incoming wave has wave vector \vec{k}_{in} and the i^{th} groove is at position \vec{R}_i , then the relative phase of the incoming wave at the groove is

$$(\phi_i)_{in} = \vec{R}_i \cdot \vec{k}_{in} \quad (4)$$

For an outgoing wave in the direction of \vec{k}_{out} , on the other hand, the relative phase of the wave emitted by the i^{th} groove is

$$(\phi_i)_{out} = -\vec{R}_i \cdot \vec{k}_{out} \quad (5)$$

The total relative phase of the i^{th} groove's contribution to the outgoing wave \vec{k}_{out} is just the sum of the phases (4) and (5). For a maximum intensity in this output direction, the contributions of each of the grooves should be in phase, so the relative phase of each groove must be an integer multiple of 2π . Since the grooves are evenly spaced along a line with separation d (figure 2), it must be true that $\vec{R}_i = i \vec{R}_1$ (i is an integer), and therefore

$$\begin{aligned} \forall i \in \mathbb{Z}, \phi_i &= (\phi_i)_{in} + (\phi_i)_{out} = \vec{R}_i \cdot (\vec{k}_{in} - \vec{k}_{out}) = i \vec{R}_1 \cdot (\vec{k}_{in} - \vec{k}_{out}) = 2\pi n_i; \quad n_i \in \mathbb{Z} \\ \therefore \boxed{\vec{R}_1 \cdot (\vec{k}_{in} - \vec{k}_{out})} &= 2\pi m; \quad m \in \mathbb{Z} \end{aligned} \quad (6)$$

Clearly, $|\vec{k}_{in}| = |\vec{k}_{out}| = 2\pi / \lambda$, so given the geometry of figure 2,

$$\lambda(\theta) = \frac{2d \cos \phi \sin \theta}{m} = \frac{2 \cos \phi \sin \theta}{mN}; \quad N \equiv 1/d \quad (7)$$

N is the groove density, which is 1200 grooves/mm for the DK-480 you will be using, and ϕ is approximately 9° . θ is adjustable through the range $0^\circ - 70^\circ$ in steps of 2.5×10^{-4} degrees. The integer m is called the **order** of the diffraction; $m=0$ corresponds to simple plane-mirror reflection; the wavelength calculated by the instrument for a given grating angle assumes that $m=1$ (first order).

Now to estimate the wavelength resolution of the monochromator: the two major effects which limit the resolution are (1) diffraction because of the finite sizes of the optical elements and (2) the angular sizes of the slit openings (slit widths) as seen by the grating.

Let's consider the diffraction limit to resolution caused by the finite size of the grating. At a given grating angle θ , then for the wavelength $\lambda(\theta)$ each groove of the grating contributes a bit that is in phase with all the others, a fact we used to derive equation (7). At a slightly different wavelength, however, there will be a phase error from each groove that varies linearly across the grating because the n_i in the first line of (6) is no longer exactly equal to an integer. Assume that the total number of grooves is M , so that the groove index i ranges from 0 to $M-1$, with, of course, $M \gg 1$. When the wavelength $\lambda' = \lambda(\theta) \pm \Delta\lambda$ is such that there is either an extra 2π or a missing 2π of phase variation across the grating, then the signal from the groove $i+M/2$ will be out of phase with the signal from groove i for each $0 \leq i < M/2$, and the output intensity should be very much less than it is at $\lambda(\theta)$. This condition on $\Delta\lambda$ implies that

$$\begin{aligned} M \vec{R}_1 \cdot (\vec{k}_{in} - \vec{k}_{out}) &= 2\pi m M (1 \pm 1/mM) \\ \frac{2d \cos \phi \sin \theta}{\lambda(\theta) \mp \Delta\lambda} &= m \left(1 \pm \frac{1}{mM} \right) = \frac{2d \cos \phi \sin \theta}{\lambda(\theta)} \left(1 \pm \frac{1}{mM} \right) \\ \therefore \frac{\lambda(\theta)}{\lambda(\theta) \mp \Delta\lambda} &= 1 \pm \frac{1}{mM} \approx 1 \pm \frac{\Delta\lambda}{\lambda(\theta)} \end{aligned}$$

Ideal grating diffraction limit to resolution:	
$\frac{\Delta\lambda}{\lambda} \approx \frac{1}{m(\text{total number of grooves})} = \frac{1}{m w N}$	(8)

where m is the order, w is the width of the grating, and N is the groove density. Because of the not-quite-perfect surfaces and alignment of the grating and additional optics, the actual resolution of the monochromator is significantly poorer than this. In first order ($m=1$) its best resolution is $\Delta\lambda \sim 0.06$ nm, but the resolution improves with order, so high-resolution measurements should be done using second or even third order.

Next consider the effect of slit opening width (refer to figure 1). A finite source slit width w_s illuminates the first focusing mirror with light coming through a small range of angles $\Delta\varphi_s = (w_s / f) \ll 1$, where f is the focal length of the mirror. Refer to figure 2 and consider the effect of rotating \vec{k}_m counterclockwise by $\Delta\varphi_s$, while leaving \vec{k}_{out} fixed (since the direction to the exit slit hasn't been changed). To restore the symmetry of the figure, we rotate the axes by $\Delta\varphi_s / 2$; this transformation changes the angles θ and ϕ to $\theta + \Delta\varphi_s / 2$ and $\phi - \Delta\varphi_s / 2$, so the new wavelength of the monochromator becomes

$$\lambda(\theta) + \Delta\lambda = \frac{2 \cos(\phi - \Delta\varphi_s / 2) \sin(\theta + \Delta\varphi_s / 2)}{m N}$$

$$\sin(\theta + \Delta\varphi_s / 2) = \sin\theta \cos(\Delta\varphi_s / 2) + \cos\theta \sin(\Delta\varphi_s / 2) \approx \sin\theta + (\Delta\varphi_s / 2) \cos\theta$$

$$\cos(\phi - \Delta\varphi_s / 2) = \cos\phi \cos(\Delta\varphi_s / 2) + \sin\phi \sin(\Delta\varphi_s / 2) \approx \cos\phi + (\Delta\varphi_s / 2) \sin\phi$$

$$\therefore \lambda(\theta) + \Delta\lambda \approx \frac{2 (\cos\phi \sin\theta + (\Delta\varphi_s / 2) (\cos\phi \cos\theta + \sin\phi \sin\theta))}{m N} = \lambda(\theta) + \frac{\cos(\theta - \phi)}{m N} \Delta\varphi_s$$

where terms of order $(\Delta\varphi_s / 2)^2$ and higher have been dropped. Similarly, a finite exit slit width w_e will accommodate a range of angles for light diffracted from the grating $\Delta\varphi_e = (w_e / f) \ll 1$, and a calculation similar to the above will result in a similar expression for its effect on the resolution. The contribution of the finite slit widths to the resolution is thus

slit width limit to resolution:

$$\Delta\lambda \approx \frac{\cos(\theta - \phi)}{m N} \left(\frac{w_s + w_e}{f} \right) \lesssim \frac{1}{m N} \left(\frac{w_s + w_e}{f} \right) \quad (9)$$

Again, m is the diffraction order and N is the groove density, as in equation (7). The slits are adjustable in the range 10 – 3000 μm . Wide slit settings let in more light, increasing sensitivity to faint lines. Wide slit settings are also very useful to reduce the spectrum resolution so that larger $\Delta\lambda$ steps may be used during wide-wavelength scans. This speeds up the scan while still detecting all lines in the spectrum.

The monochromator's light output is detected by a photomultiplier which emits a current proportional to the signal intensity. The experimenter can adjust the photomultiplier's high-voltage bias to control its sensitivity. An amplifier (with adjustable gain and low-pass filtering) converts the output current to a voltage, which is displayed on a meter on the amplifier and is also connected to a computer's data acquisition system (DAQ) analog input port AI0.

The monochromator is normally computer-controlled using an RS-232 serial connection. The program that provides the overall data acquisition and control of the experiment is ***Monochromator.exe***. The program's user interface is straightforward and includes prompts and help information. The DK-480 monochromator also includes a local, manual control panel, which allows the experimenter the opportunity to override computer control and input commands directly to the instrument.

Lenses and filters are positioned between the light source and the entrance slit to optimize the intensity at the slit and to select a particular wavelength range (so that you may determine whether a spectral line you detect is in 1st or 2nd order from the grating ($m = 1$ or $m = 2$). Most lens and filter glass is opaque to ultraviolet light, so a special *fused-silica* lens is also available to obtain good ultraviolet spectra (down to a wavelength of ~250 nm).

PRELAB PROBLEMS

1. The *Balmer Series* of hydrogen is in the visible wavelength range. It is comprised of those emission lines generated by a transition of an electron to the first excited state ($n = 2$ in equation (1)) from a higher state. If the higher state is given by $m > 2$, then show that the wavelengths of the Balmer lines are given by:

$$\frac{1}{\lambda_{m \rightarrow 2}} = \left(\frac{1}{2^2} - \frac{1}{m^2} \right) \frac{1}{\lambda_1}; \quad m \in \{3, 4, 5, \dots\}; \quad \lambda_1 = \frac{hc}{E_{\text{ground}}} \quad (10)$$

Using $\mu = m_e$ in equation (1), calculate the ground state energy E_{ground} in electron volts. This energy is called *Rydberg's Constant*. Calculate the value for λ_1 using this energy. Calculate the wavelength of the line for $m = 3$, known as *Hydrogen α* .

2. Which lines have longer wavelengths, those of hydrogen or those of deuterium? Show that for the same transition the wavelength difference between the two isotopes' lines is:

$$\lambda^H - \lambda^D \approx \frac{m_e}{m_p} \left(1 - \frac{m_p}{m_D} \right) \lambda^H \approx 0.500 \frac{m_e}{m_p} \lambda^H \quad (11)$$

where the accuracy of the equation is to one part in $m_p/m_e \approx 2000$. What is this wavelength difference if $\lambda^H = 656 \text{ nm}$ and $m_p/m_e = 1836$?

3. In this experiment you will use the **second order** diffraction from the grating to provide higher resolution. Refer to figure 2 and equation (7). What grating angle θ would give $\lambda(\theta) = 656 \text{ nm}$, near the wavelength of *Hydrogen α* ? Other parameter values you need:

$$\phi = 9^\circ$$

$$N = 1200 \text{ lines/millimeter}$$

$$m = 2 \text{ (second order)}$$

If the source and exit slits are both set to $20 \mu\text{m}$ and the monochromator focal length is 0.48 m , then what is the slit width resolution limit to $\Delta\lambda$ at this wavelength **in second order**? To what width should the source and exit slits be set to reduce the resolution to $\sim 1 \text{ nm}$?

4. The index of refraction of air reduces the wavelength of light slightly below its value in vacuum, because light travels slightly more slowly in air. If the index of refraction is n , then wavelengths are reduced by the factor $1/n$. At a temperature of 0°C and a pressure of 760 mm mercury the index of refraction of air (at $\lambda = 589 \text{ nm}$) is 1.0002926 . The quantity $(n-1)$ is proportional to the density of air. What is the index of refraction at 25°C and 740 mm mercury ?

5. **National Institute of Standards and Technology (NIST) Atomic Spectra Database.** Go to this website: <http://www.nist.gov/pml/data/asd.cfm> and familiarize yourself with how to access the database (push the “**Lines**” button). Use the lines form page (link and illustration below) to retrieve a list of the unionized mercury lines (Hg I) for wavelengths between 300 and 800 nm. What is the wavelength of the most intense Hg I line wavelength in the 500 – 600 nm range?

Make the following selections in the “Additional Criteria” section of the Lines Form:

1. Lines: Only with observed wavelengths
2. Wavelength Data: Observed (deselect Ritz)
3. Wavelengths in: Vacuum (all wavelengths)

Lines Form page http://physics.nist.gov/PhysRefData/ASD/lines_form.html

The screenshot shows the NIST Atomic Spectra Database Lines Form. The 'Additional Criteria' section is highlighted, showing the following selections:

- Lines:** All, Only with transition probabilities, Only with energy level classifications, Only with observed wavelengths
- Bibliographic Information:** TP references, Line references
- Wavelength Data:** Observed, Ritz, Observed - Ritz (difference), Wavenumber (in cm⁻¹)

PROCEDURE

NEVER EXCEED 1000V bias to the photomultiplier!

Don't touch the surfaces of the lenses and filters, because your skin oils will damage the surfaces. Don't blow on them, either.

1. Note the room temperature and pressure. Familiarize yourself with the instrument and controls. Turn on the mercury lamp. Set the photomultiplier high-voltage bias to ~500V and set the amplifier to its lowest gain setting with the low-pass filter disabled. Ensure that the amplifier signal output is connected to the computer DAQ analog input AI0. Ensure that the monochromator is in *Remote* using its control panel. Start the *Monochromator* control program on the computer.

If you are using the DK-480 manual control panel, and you inadvertently enter a command or a mode that you want to exit, try using the ← key. If this doesn't seem to help, get assistance from your TA.

Don't push the ENTER/STOP key unless you understand what it will do!

The mercury and hydrogen lamps emit near-ultraviolet light which can be harmful if you stare at them for awhile with unprotected eyes. Eyeglasses will block the UV, as will protective glasses available in the lab.

2. Install the large, visible-light (glass) lens in front of the monochromator entrance slit and rotate the filter wheel so that no filter is in the optic path. Using the control program, set the slit widths to 50 microns. Align the Hg lamp in front of the lens and focus the lamp's image on the slit. The lamp should be positioned at about twice the lens focal length from the lens (focal length = 75mm).

Now try to detect the 546.2 nm Hg line using the control program's *manual* dialog to set the wavelength. Step the wavelength using the arrow buttons until the signal indicator is maximized. If you can't find the line after a few minutes, exit the *manual* dialog and open the *scan* dialog. Perform a 2 nm scan centered at 546.2 nm with about 100 steps. The scan should find the line, and you can use the plot cursor to find the observed wavelength of the line. Reopen the *manual* dialog and set the monochromator to the line wavelength. If all else fails,

ask your TA! While watching a signal meter, either the program's or the one on the amplifier, carefully adjust the position of the Hg lamp until the signal is maximized. Lower the gain settings if necessary so that the output signal is not saturated.

Now look for the line in 2nd order. Double the line vacuum wavelength and then apply the difference between your observed line position and the actual line wavelength to this doubled value (that is, use the same wavelength *offset* that you found in your 1st order observation). Use the *manual* dialog to move to this wavelength and see if you detect the line; it will be quite a bit weaker in 2nd order. Perform a short scan to find the line if you can't find it manually. Once you've found the line, adjust the amplifier gain and the photomultiplier bias and note their effects on the signal strength.

3. Exit the *manual* dialog and open the *scan* dialog. Perform a 2 nm scan centered on the 546.2 nm line (in 2nd order) with about 100 steps. Adjust the slit widths to 20 μm and scan again. Note the effects on resolution and signal peak value. Open the slits to the width you calculated in prelab problem 4. Adjust the high-voltage bias so that the line signal is not saturated and scan again. Is the line width now approximately 1 nm?
4. Now you can increase the high voltage by another 100V – 200V and perform a scan that covers the entire visible spectrum (300nm – 700nm) **in second order** (the maximum wavelength you can set is 1500nm (a 750nm line observed in 2nd order)). The higher voltage may cause bright lines to be saturated, but will increase the sensitivity so that even weak lines will be detected. Choose the number of points so that the wavelength increment between points is about 0.7 nm (~ 1200 points?). The result will be a “catalog” of Hg lines, which you can identify using the NIST database, although **you must be careful to not confuse longer wavelength lines in 1st order with shorter wavelength lines in 2nd order.** The photomultiplier is not sensitive to wavelengths longer than ~900nm, so any lines observed at longer wavelengths must be in 2nd or higher order.

5. CALIBRATING THE MONOCHROMATOR WAVELENGTH SCALE

You will use the mercury lines to calibrate the monochromator by fitting the observed wavelength values to the values in for the various lines you have found. You must scan each line with a narrow slit width so that you can get an accurate line position. Make sure you record the expected line **vacuum** wavelengths (from the NIST database) for each line you scan (you can use the comment line in the file headers of saved scans for this, if you wish). Here is a list of a few of the prominent lines:

Useful Visual Hg Lines (Vacuum Wavelengths, nm)

365.119 (strong)	546.226 (strong)
365.588	577.120 (faint)
366.432	579.127 (very faint)
404.770 (strong)	579.227 (faint)
434.044 (faint)	580.539 (faint)
434.871	690.943 (faint)
435.955 (strong)	708.385 (faint)

Remember to observe these lines in 2nd order!

6. OBSERVING THE HYDROGEN AND DEUTERIUM LINES

Replace the Hg lamp with the hydrogen-deuterium lamp. Ask your TA how to hook it up if required. Note that there are two tubes: one containing hydrogen only, the other containing a mixture of hydrogen and deuterium. Use the hydrogen-deuterium tube. Position the tube so that you see the slit illuminated. Perform a couple of narrow scans to find the H α line pair in 1st order and then in 2nd order. Note the improved resolution in 2nd order. Estimate the separation between the hydrogen and deuterium lines. Use the *manual* dialog to center the wavelength on a line and optimize the lamp position, the high voltage, and the amplifier gain.

Open the slit widths and perform a full-spectrum scan in 2nd order. Identify the brightest hydrogen lines, which should decrease in intensity with decreasing wavelength and get closer together (Prelab Problem 1). This is the famous *Balmer Series*. Note that there seem to be many other lines in the spectrum as well, including longer wavelength lines in 1st order. Interesting...

Find and do individual precision scans of as many of the hydrogen-deuterium *Balmer* line pairs as you can find. See if you can get lines all the way up to $n = 9 \rightarrow 2$ at 383.6 nm; this may be quite challenging in 2nd order because the lines are so weak. Don't be confused by an oxygen line triplet at ~777.6 nm. You should record estimates of the line positions and the pair separation in your lab notebook for each pair of lines you find.

7. OPTIONAL OBSERVATIONS

You should have time to do one or more of these other observations:

When you first turn on the Hg lamp, it has a definite purple hue which changes to pale blue as the lamp warms up. This purple color could be due to the presence of some strong violet line or due to a mixture of blue and red lines. Identify another element in the Hg lamp. Look in the region between 700 and 900 nm. Use the amber filter to decide if the observed lines are 700–900 nm in 1st order or 350–450 nm in 2nd order. The amber filter cutoff is at 550 nm. You should use another NIST database, the atomic spectroscopic data **Finding List**, <http://physics.nist.gov/PhysRefData/Handbook/Tables/findinglist.htm> .

Look at and identify some absorption lines in sunlight. These are called the *Fraunhofer* lines. Do a broad, continuous scan of the visible spectrum to find the wavelength with maximum intensity. Use this wavelength to estimate the temperature of the sun's bright surface.

To observe near-ultraviolet lines, the glass lens must be replaced, since glass is opaque to ultraviolet light. Use the ultraviolet lens and refocus the mercury lamp on the slit. Find the Hg 546.2 nm line and optimize the lamp position and gain settings. Now scan for a line at 508 nm. Once you find the line, move to the line center using the *manual* dialog and adjust the high-voltage and gain. Note how strong this line is. Does the NIST database list a very strong Hg line at this wavelength? What color is 508 nm light? Now block the slit with the "clear" filter and note the change in the signal. Should 508 nm light be stopped by the filter? What is the actual wavelength of this line? What order are you observing it in at 508 nm? What does the database list at this wavelength? Attempt to detect this line in higher orders (it's not too hard to find it even in 5th order).

The hydrogen lamp also has some interesting near-ultraviolet features. The lamp emits spectra of water and the OH radical as well as atomic hydrogen and oxygen. The near-ultraviolet spectrum of OH (a very complicated spectrum near 308 nm) can be used to estimate the bond length and therefore the size of an atom. A brief write-up about this topic is available in the lab.

DATA ANALYSIS

What is your best estimate of the uncertainties in your Hg line positions you will use to calibrate the monochromator and how did you arrive at them? Since you have not repeated the scan of each line several times, it is impossible to determine an accurate, independent line position uncertainty for each line. Calibrate the spectral line positions by fitting the actual vacuum Hg line wavelengths versus your observed wavelengths. The slope b and intercept a should provide corrections for the observed line wavelengths.

$$\lambda_{\text{corrected}} = b \lambda_{\text{observed}} + a$$

Perform the fit **with no uncertainty values** for either the observed or atom wavelengths (all uncertainties set to 0). In this case *CurveFit* will calculate a *standard deviation* of the points from the fit, and the calculated uncertainties in the fit parameters a and b are determined by assuming that the uncertainties in the data points are given by this standard deviation. Look at the difference plot. What is the maximum error in the linear fit? Is the fractional error (error/wavelength) of the fit small? Should a nonlinear fit be used? Can the slope of the linear calibration be explained by the nonzero index of refraction of the air in the lab, or must there be some other source of error as well? Correct your observed hydrogen line wavelengths using your calibration formula. How will you incorporate the calibration uncertainties for a and b into your final results? Are these uncertainties systematic? The standard deviation of the calibration fit should be about the same as the actual uncertainties in the individual line positions.

Since the hydrogen and deuterium lines for a given transition are both observed using a single scan, does the offset parameter a affect the separation between the lines in a pair? How best should you determine a corrected value for the pair separation using your calibration? Do you know the separation with greater accuracy than you know the individual line positions? Does the line standard deviation from the calibration represent the uncertainty in the pair separation wavelength $\lambda^H - \lambda^D$?

Equation (11) provides a relation between the line separations and the hydrogen line wavelengths. Fit your corrected separations to your corrected hydrogen line wavelengths and determine the proton-electron mass ratio and its uncertainty. If the electron rest energy is 0.511 MeV (you measure this in Experiment 30a), then what is the proton rest energy? Use this value to determine the mass of a hydrogen atom and from this result estimate the value of Avogadro's number if a mole of hydrogen (H_2) masses 2 grams.