

## Experiment 20

THE GEIGER-MÜLLER DETECTOR  
and ion mobility

References	1
Introduction: the Geiger-Müller radiation detector	1
The fields in the gas and the threshold voltage	3
Total charge of the ion sheath and its initial radius	4
Dead time and recovery time	5
Quench gas	7
PRELAB PROBLEMS	8
DETECTOR SPECIFICATIONS	8
PROCEDURE	9
ANALYSIS	11

## Appendix A

## ELECTRODYNAMICS OF THE CHARGES IN THE DETECTOR

Induced charge transfer through the external circuit	A - 2
The equation of motion of the ion sheath	A - 4
Limitations of this model	A - 4

## Appendix B

## MOBILITY OF IONS IN A GAS

Statistical physics preliminaries; mean free path	B - 1
Motion of ions; mobility	B - 2
Einstein relation between mobility and diffusion	B - 3

## Appendix C

## OUTPUT SIGNAL FORMATION IN THE GEIGER COUNTER CIRCUITRY

Charge integration (CI) mode	C - 2
Pulse mode	C - 4



## REFERENCES (books available in the lab):

Knoll, *Radiation Detection and Measurement* (John Wiley and Sons: 1979): Chapter 7: “Geiger-Mueller Counters.”

Leo, *Techniques for Nuclear and Particle Physics Experiments* (2<sup>nd</sup> revised ed., Springer-Verlag: 1987, 1994): Chapter 6: “Ionization Detectors”.

Reif, *Fundamentals of Statistical and Thermal Physics* (McGraw-Hill: 1965): Chapter 12: “Elementary kinetic theory of transport processes”.

## INTRODUCTION: THE GEIGER-MÜLLER RADIATION DETECTOR

In this experiment you will investigate the properties of a *Geiger-Müller* gas ionization detector and use your data to determine the *mobility* of atomic ions in a gas as they move under the influence of an applied electric field.

The Geiger-Müller high-energy particle detector was invented by Hans Geiger, Ernest Rutherford and Walther Müller at the University of Manchester, UK, in the early part of the 20<sup>th</sup> century. Comprised of a cylindrical metal tube surrounding a very thin central wire, the interior of a modern version of the device is filled with a mixture of a noble gas (Ne) with a small amount of a halogen (Br<sub>2</sub>) (see Figure 1). Applying several hundred volts between the positive central wire (the *anode*) and the surrounding cylinder (the *cathode*) activates the detector; high-energy particles entering the detector cause current pulses in the power supply lines as the electrons and ions generated in the fill gas move toward the two conductors.

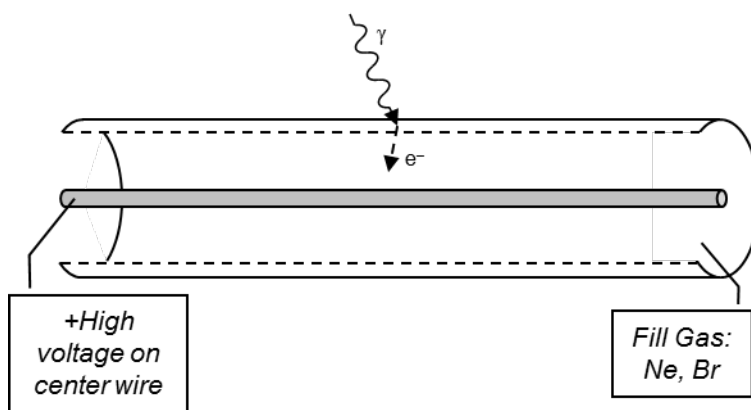


Figure 1: A sketch of a *Geiger-Müller* gas ionization detector. An energetic photon can eject an electron from the cylindrical metal tube into the gas-filled interior. The electron's acceleration toward the center wire results in a *Townsend avalanche* of electron-ion pairs and UV photons near the wire.

High energy photons entering the cylindrical wall of the detector may cause an electron to be ejected from the interior of the wall by either Compton scattering or photoelectric absorption. The electron is attracted toward the central wire because of the electric field between the wire and the detector wall. This field is very strong near the wire, and the electron may accelerate enough between collisions with the gas atoms that those collisions become increasingly violent, exciting electrons in the atoms or even ionizing them. Freed electrons cause subsequent ionizations of their own, leading to a so-called *Townsend avalanche* (c.1897 by John S. Townsend, then at Cambridge, UK) of electrons toward the central wire and an exponentially increasing concentration of ions near it, as in figure 2. The final number of electrons in a single avalanche is typically  $10^6$  or higher.

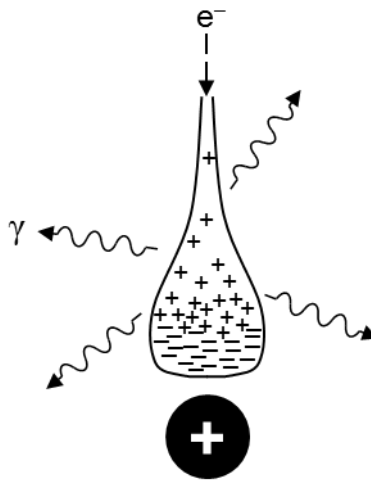


Figure 2: If the electric field near the center wire is strong enough, a free electron in the gas will cause a *Townsend avalanche* of electron-ion pairs and UV photons near the central wire of the detector. The ion concentration increases exponentially very near the wire. The UV photons trigger additional avalanches throughout the interior of the detector.

A *proportional detector* works in a similar fashion, and once a single Townsend avalanche is created the detector responds with a current pulse proportional to the charge in it. The Geiger-Müller detector goes a step further, however. Atoms excited during an avalanche decay by emitting photons which may be absorbed by other atoms throughout the interior of the detector, releasing additional electrons and triggering more avalanches. Consequently **the entire interior of the detector is quickly filled with thousands of avalanches, and the center wire is surrounded by approaching electrons and a nearby cylindrical sheath of positive ions.** Because of this spread of avalanches throughout the tube, the initial energy deposited by the original ionizing particle has a negligible effect on the total charge in the ion sheath; the Geiger-Müller detector is not capable of determining the energy of the triggering event, but its output is so large that no additional amplification of the detector's output is necessary.

The avalanches stop forming when the free electron concentration near the positively charged center wire reduces the electric field surrounding it so much that other, more remote free electrons can no longer reach high enough kinetic energies to produce additional ionizations. The entire process from initial avalanche to this complete electron+positive ion sheath formation takes about a microsecond.

## THE FIELDS IN THE GAS AND THE THRESHOLD VOLTAGE

A high voltage power supply is connected between the central wire and the shell of the detector so that electrons are accelerated toward the wire and produce the Townsend avalanches and the subsequent ion sheath. The central wire is, of course, attached to the positive terminal of the power supply so that electrons are attracted toward it. Prior to any ion formation the cylindrically-symmetric electric field between the wire and the concentric conductive tube (except near the ends of the tube) is easy to derive and is given in equation (1).

$$\vec{E}_0(\vec{r}) = \frac{1}{2\pi\epsilon} \frac{q_0}{L} \frac{\hat{r}}{r} \quad (1)$$

where  $q_0$  is the initial positive charge on the center wire and  $L$  is its length. Experiments have determined that an electron needs a kinetic energy of  $\sim 36$  eV on average to ionize a neon atom (see references). In the detector you will use, fields on the order of  $\sim 10^5$  Volts/meter are required for electrons to achieve this energy and initiate or continue an avalanche. The line integral of the field from the surface of the center wire (radius  $a$ ) to the tube (inside radius  $b$ ) gives the potential difference, which is the power supply bias voltage  $V_0$  when no ions are present:

$$V_0 = \frac{1}{2\pi\epsilon} \frac{q_0}{L} \ln(b/a); \quad \therefore E_0(r) = \frac{V_0}{r \ln(b/a)} \quad (2)$$

The minimum value of  $V_0$  for which the detector will produce an ion sheath is called the *Threshold voltage*,  $V_{Th}$ , and the corresponding electric field strength at the surface of the center wire is  $E_{Th}$ . The violence of the avalanches and thus the charge density in the resulting ion sheath increase roughly linearly as the bias voltage is increased beyond  $V_{Th}$ .

If the bias voltage becomes too high, the electric field is so strong that thermal collisions in the gas may ionize an atom and spontaneously initiate an avalanche. At this point the detector continually generates false detections and may even experience electric discharges (sparks) within the tube. **Operating the detector at such a high voltage will very quickly cause permanent damage to it. For the detectors you will use, this limit is reached at approximately 1100V.**

## TOTAL CHARGE OF THE ION SHEATH AND ITS INITIAL RADIUS

As the bias voltage applied to the detector is increased above  $V_{Th}$  the average total charge in the positive ion sheath increases as well, as mentioned previously. The positive ions are highly concentrated near the center wire, as discussed in Appendix A to this experiment. The mean radius of this initial ion sheath and its total charge determine the subsequent dynamics of the free electrons and the positive ions as described in that Appendix.

The total charge of the positive ion sheath may be determined if the external Geiger counter circuitry is configured in *Charge Integration Mode* (see Appendix C to this experiment). Figure 3 shows screen captures of an oscilloscope recording of the output in this mode; the total charge in the ion sheath ( $q_p$ ) is proportional to  $V_p$ , the maximum output voltage of a pulse, which occurs when the ions and electrons have all reached their respective conductors in the detector (outer shell for the ions and center wire for the electrons). The left-hand image in Figure 3 shows that  $q_p$  at 850V bias is approximately 60% larger than at 800V for the detector used.

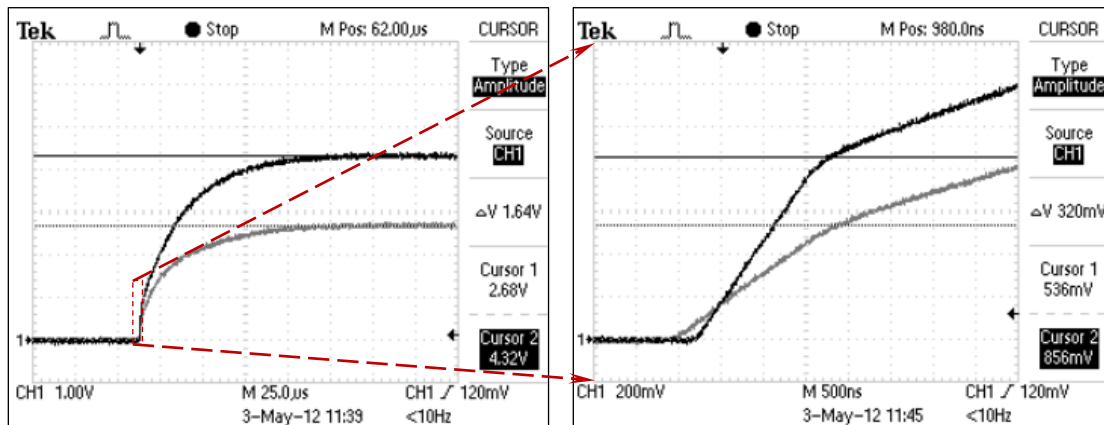


Figure 3: Oscilloscope screen-captures of the output in *Charge Integration Mode* for two different detector bias voltages: 800V and 850V. The image on the left shows the maximum output voltages for events detected for each of the bias voltages; as explained in Appendix C, the maximum output voltage,  $V_p$ , is proportional to the total charge in the ion sheath. The right-hand image shows the initial rise of detection events at each of the two bias voltages, as indicated by the dashed box and arrows. The initial, steep, linear portion (ending at voltage  $V_e$ , where a cursor is positioned) gives estimates of the time required for ion sheath formation and its initial radius,  $r_{s0}$ .

The initial portion of an output pulse in *Charge Integration Mode* (right-hand image in Figure 3) rises very rapidly as the ion sheath is formed and the free electrons are collected at the center wire. It is evident from the figure that ion sheath formation requires less than  $\sim 2 \mu\text{sec}$  for each of the events shown. The voltage at the end of this initial stage,  $V_e$ , may be used to estimate  $q_e$  (see Appendices A and C of the experiment). The ratio  $V_e / V_p$  may then be used to estimate the initial ion sheath radius (equation (C8)). The  $dV/dt$  slope just beyond  $V_e$  may be used to estimate the  $\text{Ne}^+$  mobility,  $\mu$  (see (C9) and Appendix B).

## DEAD TIME AND RECOVERY TIME

The modification to the field following an ionization event and the subsequent motions of the free electrons and ions are investigated in Appendix A to this experiment. Once an ionization event occurs and an ion sheath is formed around the center wire, the free electrons quickly gather around the center wire causing the electric field in the detector to drop too low for another such event to occur. The field recovers as the positive ion sheath moves away from the center wire, returning to the initial value (2) when the positive ions have all reached the outer conductive shell (the cathode); Figure 4 illustrates this behavior.

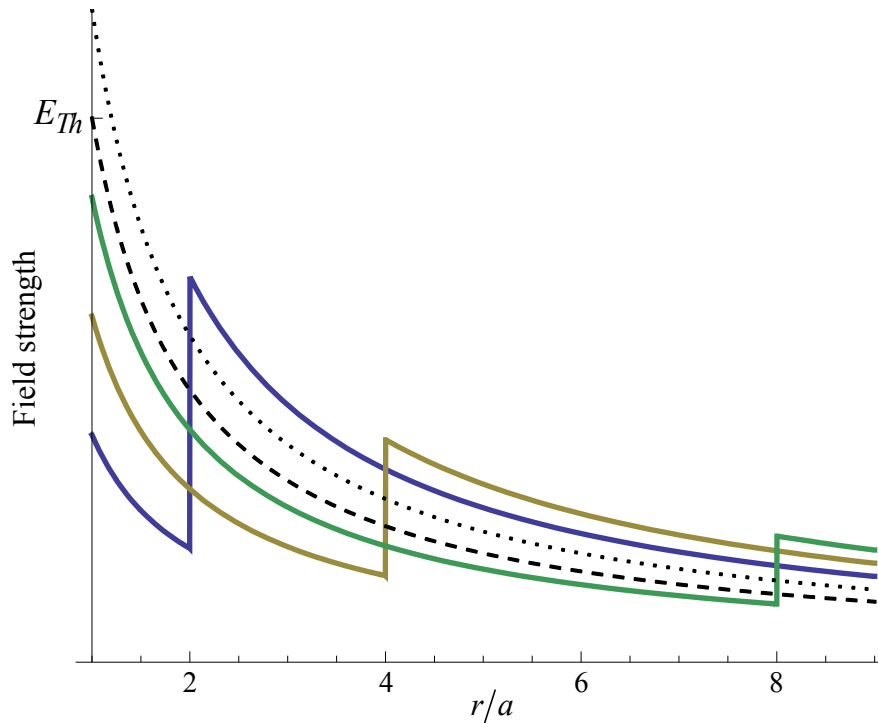


Figure 4: Effect of the free electrons and a thin positive ion sheath on the electric field. The electrons have gathered at the center wire (radius =  $a$ ), reducing the field there below the threshold field strength. As the ion sheath moves outward, the induced charge on the center wire increases the field back toward its initial value,  $E_0$  (dotted line). When the field strength exceeds  $E_{Th}$  (dashed line), new events may be detected. Ion sheath positions of 2, 4, and 8 times  $a$  are shown; the field reaches  $E_{Th}$  when the ion sheath reaches  $13a$  for the hypothetical case illustrated here ( $E_0 = 1.2 E_{Th}$ ).

If  $V_0 > V_{Th}$ , then at some time before full recovery of its initial strength the field will become strong enough for new avalanches to be generated, though with reduced strength. This occurs when the electric field at the surface of the center wire has recovered to at least  $E_{Th}$  (see Figure 4). Until this happens, the detector cannot respond to a new high-energy particle interaction. The time it takes for the field to recover to  $E_{Th}$  is called the *dead time*,  $t_D$ . The additional time required for full recovery to  $E_0$  is called the *recovery*

time,  $t_R$ . This phenomenon may be observed by looking at output pulses generated by the detector when exposed to a strong source of ionizing photons (how short output pulses are produced by the external circuitry is described in the *Pulse Mode* section of Appendix C).

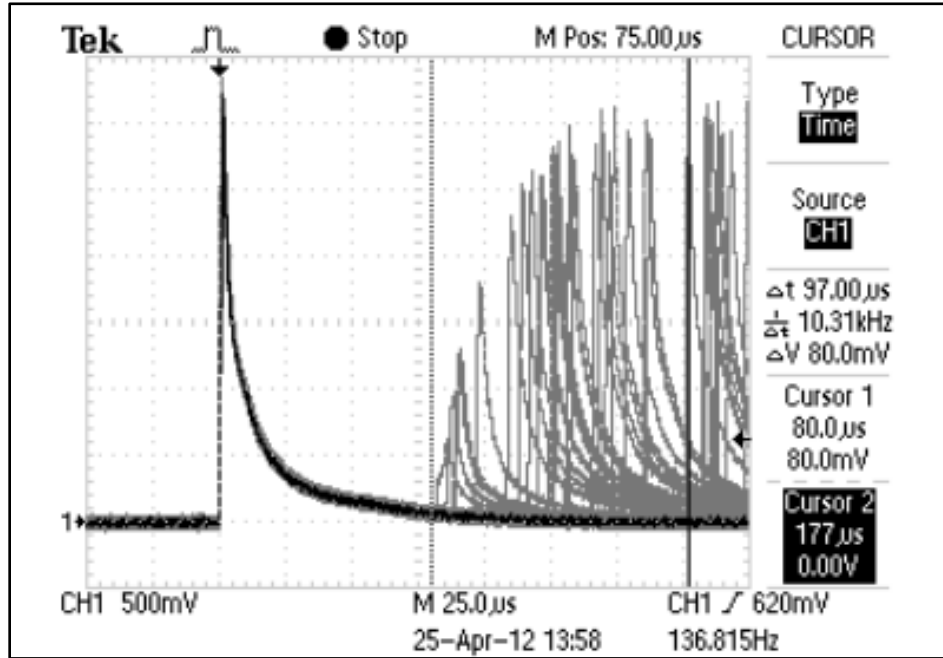


Figure 5: Oscilloscope screen-capture of many superimposed pairs of closely-spaced detector output pulses. Pulse height varies with the strength of the Townsend avalanches for that detection event. Cursor 1 is positioned  $t_D$  (dead time) after the first pulse of a pair. Cursor 2 is positioned close to  $t_R$  (recovery time) after Cursor 1.  $V_0 = 805\text{V}$  for this data.

Figure 5 shows a screen capture of an oscilloscope recording of many pairs of pulses occurring close together, with the external circuit configured in *Pulse Mode* (Appendix C). Note from the figure that no pulses are visible until 80 microseconds have elapsed following the first pulse, so  $t_D = 80\ \mu\text{sec}$ . Pulses arriving shortly after  $t_D$  are very small, corresponding to internal field strengths close to  $E_{Th}$ . Approximately another 97 microseconds must elapse before a second pulse will be as intense as the first ( $t_R \approx 97\ \mu\text{sec}$ ), a total of  $177\ \mu\text{sec}$  following the first pulse. It should be clear from the figure that although  $t_D$  may be straightforward to measure, an accurate determination of  $t_R$  may be difficult because of the slowly rising pulse amplitude as  $t_R$  is approached.

The speed with which the ion sheath moves under the influence of the electric field is determined by the ions' mobility, as defined by (A9) and examined in detail in Appendix B to this experiment.



## QUENCH GAS

The gas in the detector is a mixture of neon and a few percent bromine. Since neon is the primary fill gas, most of the positive ions in the initial sheath are also neon. If neon were the only constituent of the gas, then these ions would reach the cathode (stainless steel) and be neutralized by an electron transfer from the cathode. The ionization potential for Ne is 21.6eV, and the work function for stainless steel is 4.4eV. Consequently, the electron transfer is exothermic, releasing 17.2eV/ion. Since this energy is  $\gg 4.4\text{eV}$ , it is possible that the energy release may cause the emission of an extra electron from the cathode surface. That electron would then be accelerated toward the center wire, leading to another Townsend avalanche and the subsequent formation of another ion sheath! This process could be repeated indefinitely, leading to a continual output of pulses from the detector.

Although the probability is small that a neon ion neutralization at the cathode will lead to an additional electron release, there are very many ions in the sheath, especially for higher bias voltages. Thus the resulting probability of at least one electron release can be quite high. To prevent this multiple (or even continual) pulse generation triggered by a single event, a *quench gas* is added to the tube. A halogen such as bromine has a much lower ionization energy than neon (11.8eV). The probability that bromine ion neutralization at the cathode will lead to an additional electron emission is acceptably small, and little multiple pulse generation occurs.

As the neon ions of the initial sheath formation drift toward the cathode, they collide with many bromine molecules, and these collisions eventually result in an electron transfer from the bromine to the neon, converting the neon ions to bromine ions during the sheath's drift toward the cathode. As a result the sheath is nearly all  $\text{Br}^+$  when it reaches the cathode.

## PRELAB PROBLEMS

Use the detector specifications listed below for these problems. You also need to have read this experiment's Appendices (A – C) to answer these problems.

1. Determine the detector's capacitance in picoFarads (neglecting end effects). Assume that the fill gas permittivity is the same as that for vacuum.
2. Refer to the data in Figure 3. Which of the pulses (dark or light gray) corresponds to the higher bias voltage (850V)? Using the cursor voltage values shown in the figure (corresponding to  $V_p$  and  $V_e$  of Appendix C) calculate the positive ion sheath's initial radius ( $r_{s0}$ ) for each of the two bias voltages.
3. Calculate the charges  $q_0$ ,  $q_p$ , and  $q_e$  from the data in Figure 3 for the 850V bias pulse. Assume the circuit capacitance is 1.1 nF and the detector capacitance is 2.6 pF. Is inequality (A17) satisfied? What does this imply about the thin sheath approximation used in Appendix A for this case?
4. What would be the threshold voltage,  $V_{Th}$ , if the threshold electric field at the surface of the center wire for ion sheath formation ( $E_{Th}$ ) is 7500 Volts/centimeter?
5. What is ratio of the mobility of an ion in the detector to its mobility at the standard pressure of 760 Torr (Table B-1)? Assume the detector gas is at 25°C. What would you expect the measured mobility (in  $\text{cm}^2/\text{V}/\text{sec}$ ) of  $\text{Ne}^+$  to be in the detector?

## DETECTOR SPECIFICATIONS

Center wire radius	$a$	0.3175 mm = 0.0125 inch
Shell radius	$b$	7.62 mm = 0.300 inch
Length	$L$	14.76 cm = 5.81 inch
Fill gas pressure	$p$	425 Torr @ 25° C
Gas composition		98% Ne + 2% Br <sub>2</sub>

## PROCEDURE

During this experiment you will:

1. Examine the variation in the ion sheath charge with detector bias voltage,  $V_0$ , and estimate the threshold voltage,  $V_{Th}$ , of a Geiger-Müller detector.
2. Estimate the variation in ion sheath initial formation radius,  $r_{s0}$ , with  $V_0$ .
3. Examine the variation in detector dead time,  $t_D$ , with bias voltage.
4. Estimate the mobility,  $\mu$ , of  $\text{Ne}^+$  ions in the detector.

**You must have the most recent version of *CurveFit*, which has been upgraded to include a function for loading the oscilloscope data format.**

### *Detector Circuit*

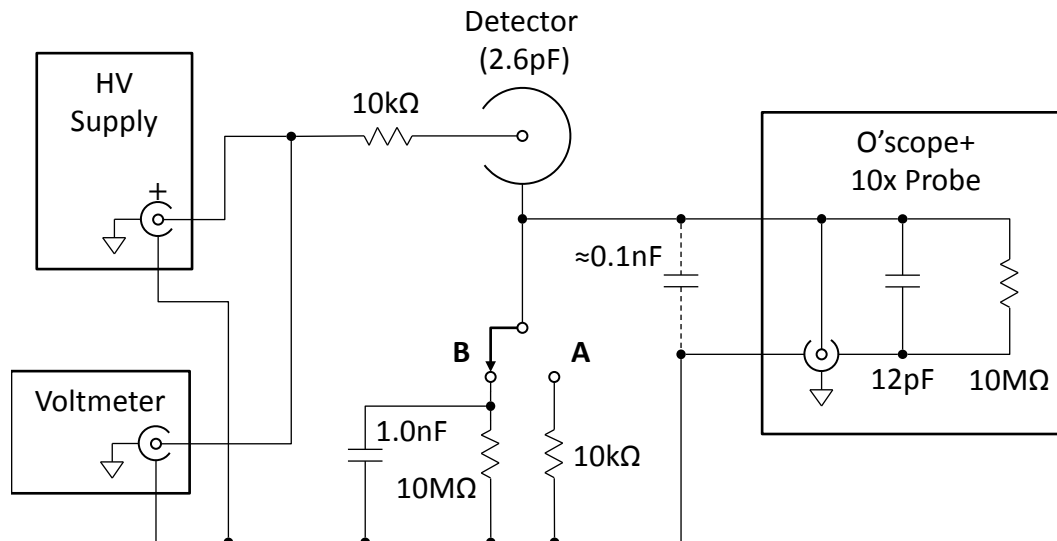


Figure 6: The circuit used for the experiment. Switch position A puts the system in *Pulse Mode*, and position B puts it in *Charge Integration (CI) Mode* (see Appendix C). A total circuit stray capacitance of  $\sim 0.1\text{nF}$  is also indicated. The  $10\text{k}\Omega$  resistor in series with the detector anode (center wire) is to limit current flow and protect you from electric shocks.

Figure 6 is a schematic diagram of the circuit you will use. The oscilloscope must be connected to a computer so that waveforms may be saved.

**Never exceed 1100V or the detector may be permanently damaged by continuous discharge.**

### ***Initial Setup***

Start with a bias voltage of +800V to +850V, and with a 10x probe connecting the circuit output to the oscilloscope input. After setting the trigger, time/division, and volts/division on the oscilloscope appropriately, you should see occasional pulses from background radiation. In *Charge Integration (CI) Mode* (circuit switch position B, 10M $\Omega$ ) you should see pulses similar to those in the left image of Figure 3. In *Pulse Mode* (switch position A, 10k $\Omega$ ) you should see pulses similar to those in Figure 5. Make sure the oscilloscope's **CH 1 MENU** shows that a 10x probe is being used, so displayed voltages are correct.

Other useful oscilloscope buttons and options:

**RUN/STOP** Push this button to pause and hold a particular pulse for analysis or resume triggering.

**SINGLE SEQ** Push this button to have the oscilloscope capture a single pulse and then pause. Use **RUN/STOP** to resume normal, continuous operation.

**DISPLAY** Push this button to call up the DISPLAY menu on the screen. The PERSIST menu choice may be set to **Infinite** to hold all acquired sweeps and generate a display of multiple pulses as in Figure 5.

**CURSOR** Push this button to call up the CURSOR menu on the screen and use the cursors to make measurements.

Start the *OpenChoice Desktop* software, and select the USB instrument (which should be the oscilloscope). Attempt a *Screen Capture* and a *Waveform Data Capture* using the software. The screen capture image may be saved to disk in any of a number of image file formats. The waveform data may be saved in either CSV or Text (tab delimited) format for later import into *CurveFit*.

### ***Charge Integration Mode Measurements***

The pulse maximum output voltage in CI mode ( $V_p$ ) provides a measurement of the total charge in the positive ion sheath,  $q_p$ . This charge approaches 0 as the bias voltage is reduced toward the threshold voltage,  $V_{Th}$ . Measure  $V_p$  as you reduce the bias voltage,  $V_0$ , from 800V. You should get several data points down to at least 760V. Estimate  $V_{Th}$  by reducing  $V_0$  to the point where the pulses just vanish. You will determine  $V_{Th}$  more accurately during your data analysis.

What is the total charge on the center wire ( $q_0$ ) at your estimated  $V_{Th}$ ? Use (A17) to estimate the maximum ion sheath charge  $q_p$  for which the analysis of Appendix A is valid. What is then the maximum pulse output voltage corresponding to this charge?

Capture waveform data for at least three different bias voltages which result in  $V_p$  values of 2 Volts or less. Make sure you perform two captures for each  $V_0$ : one with a time scale

long enough to see the peak voltage and one with a much shorter time scale and higher vertical sensitivity so that you can accurately determine  $V_e$  and  $dV_{\text{out}}/dt$  just above  $V_e$  (similar to the two images in Figure 3). You will use this data to determine the mobility of the  $\text{Ne}^+$  ion.

Record  $V_p$  vs  $V_0$  data for several voltages between 800V and 950V. Finally, increase the time/division enough to see the exponential decay in the CI pulse output (as in Figure C2). Capture this data for later determination of the decay time constant using *CurveFit*.

### ***Dead Time Measurements***

Change the detector circuit to *Pulse Mode* and set  $V_0$  to  $\sim 850\text{V}$ . Use a  $^{137}\text{Cs}$  source set just under the detector. Adjust the oscilloscope to see an output similar to Figure 5, and then set the display persistence to infinite. Let the display build for several minutes, until you can accurately determine the dead time (you should have many more pulses than are shown in Figure 5). Position a cursor at the dead time position ( $t_D$ ) and save an image of the oscilloscope screen. Repeat this for at least 3 other bias voltages between 780V and 950V.

### ANALYSIS

A linear fit of your  $V_p$  vs  $V_0$  data for bias voltages of 800V and less should allow you to determine  $V_{Th}$ . Should you fit with  $V_0$  as  $x$  or as  $y$  if the  $y$ -intercept of the linear fit is to be  $V_{Th}$ ? Does the relationship between  $V_p$  and  $V_0$  remain linear over the entire 760V to 950V bias range?

Use your long, exponential decay data of the CI pulse to determine the  $RC$  time constant  $\tau$  of the circuit in CI mode (it should be  $\sim 5$  or  $6$  ms).  $R = 5\text{M}\Omega$  (two  $10\text{M}\Omega$  in parallel); what is the total circuit  $C$ ? You will need this value to use with equation (C9).

Estimate  $r_{s0}$  values from your  $V_p$  and  $V_e$  data and the dimensions  $a$  and  $b$  of the detector. How does  $r_{s0}$  vary with  $V_0$ ? Remember,  $V_e$  is the voltage where you see an abrupt change in the slope of  $V_{\text{out}}$  only a few  $\mu\text{sec}$  following the start of the pulse. Below  $V_e$  you should note that  $V_{\text{out}}$  vs time is remarkably linear.

Perform a linear fit to a portion of the data below your estimate of  $V_e$  and another linear fit to the data just above your estimate; the intersection of the two fit lines should be a better determination of  $V_e$  (if your two linear fits are  $V = a_1 + b_1 t$  and  $V = a_2 + b_2 t$ , then  $V_e = (a_1 b_2 - a_2 b_1) / (b_2 - b_1)$ ). Now you can get a more accurate calculation of  $r_{s0}$  using equation (C8), and the slope of the second fit provides the  $dV_{\text{out}}/dt$  value you need for equation (C9) and a determination of the mobility of  $\text{Ne}^+$ ,  $\mu$ . Convert your mobility to a value for 760 Torr pressure and compare to the value in Table B-1. Repeat for your other data at different bias voltages.

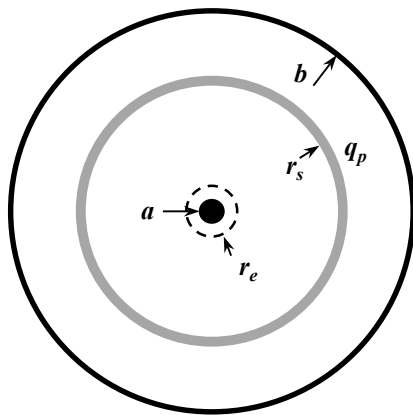


## APPENDIX A

## ELECTRODYNAMICS OF THE CHARGES IN THE DETECTOR

If the voltage difference between the center wire and the surrounding metal tube of a Geiger-Müller detector is large enough, a high-energy particle may initiate a Townsend avalanche, culminating in the formation of a cylindrical sheath of free electrons and their parent ions surrounding the center wire (as described in the Introduction). Before such an event occurs the gas filling the detector contains no free charges, and the electrostatic field in the gas is described by equations (1) and (2) of the main text, where  $V_0$  is the initial potential established across the detector by the high-voltage power supply, and  $q_0$  is the initial positive charge on the center wire.

Once a particle is detected electron and ion sheaths are established very close to the center wire, where the electric field is strongest. Because the ion formation increases exponentially as an avalanche approaches the center wire (Figure 2 of the main text) the effective thickness of the newly-formed positive ion sheath may be assumed to be quite small compared to its mean radius. We thus assume that it may be modeled as a thin, uniform shell of charge surrounding the center wire at radius  $r_s$  with total charge  $q_p$ . For the theory presented here we assume that this shell of charge remains thin as its radius expands toward the outer conductor. The conditions required for the shell to remain thin are investigated later. For the derivations which follow we assume that the free electrons in the gas also form a thin shell as they fall toward the center wire, but the results will be the same whether or not this is true. The charge configuration and the associated variables are defined in Figure A-1.



- $a, b$  : radii of the anode (center wire) and the cathode
- $r_s$  : radius of the positive ion sheath (initially  $r_{s0}$ )
- $r_e$  : radius of the free electron sheath (initially  $r_{e0} = r_{s0}$ )
- $q$  : charge on the anode (initially  $q_0$ )
- $q_p$  : total charge of the positive ion sheath
- $V$  : potential of the anode wrt the cathode (initially  $V_0$ )
- $L$  : length of the detector ( $q / L$  is the linear charge density)

Figure A-1: A schematic cross-section through the detector showing the positive ion sheath and the free electrons. The symbols used in the derivations of this section are also defined here. The total charge of the free electrons is, of course,  $-q_p$ .

The cylindrically-symmetric electric field at any point within the detector is easy to derive by considering the electric flux through the surface of a cylinder of radius  $r$ :

$$\vec{E}(\vec{r}) = \frac{1}{2\pi\epsilon} \frac{q_{inside}}{L} \frac{\hat{r}}{r} \quad (\text{A1})$$

where  $q_{inside}$  is the total charge inside radius  $r$ .

Using (A1) and the fact that the total charge of the free electrons is  $-q_p$  (charge conservation!), it is evident that the electric field outside of the volume between the electron and ion sheaths is determined solely by the charge on the center wire,  $q$ . Between the two sheaths the electric field is reduced by the presence of the electrons' charge  $-q_p$ . The potential difference between the anode and cathode is thus given by (A2).

$$\begin{aligned} V &= \int_a^b \vec{E}(\vec{r}) \cdot d\vec{r} = \frac{1}{2\pi\epsilon} \left( \frac{q}{L} \int_a^{r_e} \frac{dr}{r} + \frac{q - q_p}{L} \int_{r_e}^{r_s} \frac{dr}{r} + \frac{q}{L} \int_{r_s}^b \frac{dr}{r} \right) \\ &= \frac{1}{2\pi\epsilon} \left( \frac{q}{L} \ln \frac{b}{a} - \frac{q_p}{L} \ln \frac{r_s}{r_e} \right) \end{aligned} \quad (\text{A2})$$

The electrons' mobility in the fill gas is  $\sim 10^3$  times that of the much larger, heavier ions, so the electrons freed in any particular avalanche fall quickly toward the center wire before their associated ions can move a significant distance away from their initial positions. All of the electrons fall to the center wire surface (at radius  $a$ ) as the entire ion sheath forms over a period of  $\sim 1$  microsecond. From this time onward  $r_e = a$  in equation (A2), whereas  $r_s \rightarrow b$  over the next  $\sim 10^2$  microseconds. During this latter period of ion motion the electric field near the center wire evolves as:

$$E(a) = \frac{1}{2\pi\epsilon} \frac{q - q_p}{aL} \quad (\text{for } r_e = a) \quad (\text{A3})$$

## INDUCED CHARGE TRANSFER THROUGH THE EXTERNAL CIRCUIT

To proceed further one must determine the function  $q(r_s)$  for the charge on the center wire using (A2). This function depends on the details of the external circuit connected to the detector (through the impressed voltage  $V = V(q, r_s)$ ), but consider the simple case where the entire external circuit consists of just the power supply, so the potential across the detector is always fixed at  $V_0$ .

Before the formation of the ion and electron sheaths, the potential and charge are related by equation (2). This relation is conveniently described using the definition of the *capacitance* of the detector,  $C_d$ :

$$q_0 = C_d V_0, \quad C_d = 2\pi\epsilon L / \ln(b/a) \quad (\text{A4})$$



For the detector you will use  $C_d \approx 2.6$  pF (pico Farad). Since we assume that  $V = V_0$ , equation (A2) becomes:

$$V_0 = \frac{1}{C_d} \left[ q - q_p \frac{\ln(r_s/r_e)}{\ln(b/a)} \right] \quad (\text{A5})$$

$$q - q_0 = q_p \frac{\ln(r_s/r_e)}{\ln(b/a)} \quad (\text{for } V = V_0, \text{ a constant})$$

Note that (A5) implies that charge  $q - q_0$  is transferred through the external circuit to the center wire by the power supply as the ion and electron sheaths move through the interior of the detector, even though neither sheath may have reached its respective terminal! This *induced charge transfer* from the cathode to the anode as the charges move through the gas ensures that once  $r_e = a$  and  $r_s = b$ , the power supply has already transferred a total charge of  $q_p$  to cancel the charge transfer within the gas, restoring the initial state of total charge  $q_0$  on the center wire ( and, of course,  $-q_0$  on the external conducting tube).

We can thus measure the total ion charge  $q_p$  by measuring the total induced charge transferred through the external circuit,  $q - q_0$ . In the first microsecond or so of this transfer, the induced charge is overwhelmingly determined by the free electrons moving from the ion sheath initial radius  $r_{s0}$  to the surface of the wire at  $a$ . Once this initial electron motion is finished, a much slower induced charge transfer takes place as the heavy ions move outward. If we call the induced charge transfer due the initial electron motion  $q_e$ , then from (A5)

$$q_e = q_p \frac{\ln(r_{s0}/a)}{\ln(b/a)} \quad (\text{A6})$$

We can therefore estimate the initial radius of the thin ion sheath from measurements of  $q_e$  and  $q_p$ :

$$\boxed{r_{s0} = a (b/a)^{q_e/q_p}} \quad (\text{A7})$$

Following the free electron collection at the center wire, equation (A1) determines the electric fields at the inside and outside surfaces of the positive ion sheath:

$$\vec{E}(r_s <) = \frac{1}{2\pi\epsilon} \frac{q - q_p}{L} \frac{\hat{r}}{r_s} = \frac{1}{2\pi\epsilon L} \left[ q_0 + q_p \left( \frac{\ln(r_s/a)}{\ln(b/a)} - 1 \right) \right] \frac{\hat{r}}{r_s} \quad (\text{A8})$$

$$\vec{E}(r_s >) = \frac{1}{2\pi\epsilon} \frac{q}{L} \frac{\hat{r}}{r_s} = \frac{1}{2\pi\epsilon L} \left( q_0 + q_p \frac{\ln(r_s/a)}{\ln(b/a)} \right) \frac{\hat{r}}{r_s}$$

## THE EQUATION OF MOTION OF THE ION SHEATH

The ions move away from the center wire under the influence of the electric field in the gas. The ion *mobility*,  $\mu$ , determines the average velocity at which the ions move (see Appendix B):

$$\vec{v}_{ion} = \mu \vec{E}_{ion} \quad (A9)$$

The effective electric field which determines the motion of a thin ion sheath is the mean of the fields on either side of it, equations (A8), so  $\vec{E}_{eff} = \frac{1}{2} \vec{E}(r_s <) + \frac{1}{2} \vec{E}(r_s >)$ , and:

$$\frac{d}{dt} r_s(t) = v_{ion} = \mu E_{eff}(r_s) = \mu \frac{1}{2\pi\epsilon L} \left[ q_0 + q_p \left( \frac{\ln(r_s/a)}{\ln(b/a)} - \frac{1}{2} \right) \right] \frac{1}{r_s} \quad (A10)$$

Equation (A10) can be integrated formally to provide the time at which the sheath reaches  $r_s$ , starting from its initial radius near the center wire,  $r_{s0}$ :

$$t(r_s) = \frac{1}{\mu} \int_{r_{s0}}^{r_s} \frac{dr'_s}{E_{eff}(r'_s)} \quad (A11)$$

Given equations (A10) and (A7) a numerical integration of (A11) is now possible, giving the time evolution of the ion sheath's position. The time evolution of the induced charge transfer through the circuit is then, from (A5):

$$q(t) - q_0 = q_p \frac{\ln(r_s(t)/a)}{\ln(b/a)} \quad (A12)$$

The initial velocity of the positive ion sheath (when it is at  $r_{s0}$ ) is, using (A6) and (A10), and the resulting rate of change of  $q(t)$ , using (A12), are:

$$\left. \frac{dr_s}{dt} \right|_{r_{s0}} = \mu \frac{1}{2\pi\epsilon L} \left( q_0 + q_e - \frac{1}{2} q_p \right) \frac{1}{r_{s0}} \quad (A13)$$

$$\left. \frac{dq}{dt} \right|_{r_s=r_{s0}} = \frac{q_p}{\ln(b/a)} \frac{1}{r_{s0}} \left. \frac{dr_s}{dt} \right|_{r_{s0}} \quad (A14)$$

## LIMITATIONS OF THIS MODEL

For the entire thickness of the ion sheath to move away from the center wire, it must be true that the electric field at the sheath's inner surface points away from the wire, so, from (A8), it must be true that

$$q_0 + q_e > q_p \quad (A15)$$

What if (A15) is not satisfied, which may often be the case for  $V_0 \geq 1.2V_{Th}$ ? In this case not all electrons will immediately fall to the center wire's surface, because the field near the inner surface of the sheath will disappear when the center wire's charge has been cancelled by only a fraction of the total number of free electrons available. The outer edge of the ion sheath will still expand away from the center wire, however, because of the mutual repulsion of the ions in the sheath. Equation (A5) indicates that this expansion will lead to additional induced charge on the center wire, attracting more of the electrons to it as the sheath expands. In addition, significant electron-ion recombination may occur near the inner surface of the sheath, reducing  $q_p$  to the point that (A15) becomes satisfied. Accurate modeling of these processes is potentially difficult and beyond the scope of this text, but this qualitative description indicates that the total charge  $q_p$  of the ion sheath will not increase indefinitely as  $V_0$  is increased well beyond  $V_{Th}$ .

One implication of this argument is that for high values of  $V_0$  it may be that  $q_0 + q_e \approx q_p$ , so that (A6) or (A7) defines a unique relationship between  $q_p$  and  $r_{s0}$  for a given  $V_0$ .

Even if (A15) is satisfied, the thickness of the ion sheath increases as the sheath moves outward because of the mutual repulsion of its constituent ions. The difference in the electric fields on the two surfaces of the sheath determines the rate of growth of its thickness; the ratio of this growth to the sheath expansion rate (A10) gives the change in sheath thickness for a change in its radius.

Using equations (A8), and defining  $\Delta$  to be the ion sheath thickness, with  $\Delta \ll r_s$ :

$$\begin{aligned} \frac{d\Delta}{dt} \propto |\bar{E}(r_{s>}) - \bar{E}(r_{s<})| \propto q_p; \quad \frac{dr_s}{dt} \propto \frac{1}{2} |\bar{E}(r_{s>}) + \bar{E}(r_{s<})| \propto q - \frac{1}{2} q_p \\ \therefore \frac{d\Delta}{dr_s} = \frac{q_p}{q - \frac{1}{2} q_p} \end{aligned} \quad (\text{A16})$$

The growth of the relative thickness of the sheath is most rapid while it is near the center wire, since  $q$  is smallest at the start of the sheath's motion. Once the electrons are collected,  $q > q_0 + q_e$ , so the sheath will remain relatively thin only if

$$q_0 > \frac{3}{2} q_p - q_e \approx \frac{3}{2} q_p \quad (\text{A17})$$

Because the capacitance  $C_d$  of the detector is small, the initial charge  $q_0$  may be quite small, even for a large supply voltage  $V_0$ . For the self-limiting case discussed in connection with (A15), where  $q_0 + q_e \approx q_p$ , (A16) implies that the sheath thickness grows twice as fast as the sheath radius, and the thin sheath approximation is certainly invalid.



**APPENDIX B**  
**MOBILITY OF IONS IN A GAS**

In this appendix we consider the average velocity with which an ion will drift in a gas when exposed to an electric field. This problem is very similar to that of electrical conduction in solids, as you will recognize if you performed Experiment 24, *Temperature Coefficient of Resistivity*, in Physics 6. It is also related to the discussion of General Appendix B, *Cross Sections*, and we assume that you have already reviewed that appendix.

STATISTICAL PHYSICS PRELIMINARIES; MEAN FREE PATH

The Geiger-Müller detector contains a gas mixture of Ne and Br<sub>2</sub> at a pressure  $P = 425$  Torr (at 25°C). Assuming the gas is *ideal*, (a very accurate assumption in this case), the equation of state is:

$$n = P / kT \quad (\text{B1})$$

where  $n$  is the number density of the particles in the gas, and  $k$  is Boltzmann's constant. **This number density will not vary with the lab room temperature** because the volume of the detector is fixed. For the fill pressure and temperature specified,

$$n \approx 1.4 \times 10^{19} / \text{cm}^3 \quad (\text{B2})$$

The gas is, of course, in thermal equilibrium at the lab room temperature,  $T$ . Each atom, molecule, ion, or free electron in the gas will fly about with its own random, thermal velocity, colliding with other particles in the gas and exchanging energy and momentum. As we know from the kinetic theory of gasses, the mean kinetic energy of a particle in each of its 3 spatial degrees of freedom ( $\hat{x}, \hat{y}, \hat{z}$ ) is  $\frac{1}{2} kT$ , and the velocity distribution of the particle in each dimension is Gaussian with mean = 0 (no electric field) and  $\sigma_v = \sqrt{kT/m}$ . The average 1- and 3-dimensional speeds of a particle in the gas are thus

$$\begin{aligned} \text{1 dimension: } (v_x)_{\text{RMS}} &= \sqrt{kT/m}; & \bar{v}_x &= \sqrt{(2/\pi)kT/m} \\ \text{3 dimensions: } v_{\text{RMS}} &= \sqrt{3kT/m}; & \bar{v} &= \sqrt{(8/\pi)kT/m} \end{aligned} \quad (\text{B3})$$

These speeds vary from  $\sim 200$  m/s for bromine to  $\sim 560$  m/s for neon to  $\sim 10^5$  m/s for free electrons. These speeds are several times faster than the average ion drift velocities in the detector due to the electric field, except very near the center wire.

The mean distance that a particle travels between collisions in a gas is called the *mean free path*,  $\lambda$ . The probability that a particle will travel at least a distance  $r$  from its current position before suffering another collision is independent of how far it has travelled up to that point and is given by:

$$P(r' > r) = \exp(-r/\lambda) \quad (\text{B4})$$

Compare this description of the probability of avoiding an interaction with another particle in the gas to the discussion in General Appendix B leading up to its equation (B-12). Comparing that equation with (B4), it is clear that for high-speed particles traveling through a medium (so that the target particles of the medium can be thought of as motionless),  $\lambda = (n\sigma_S)^{-1}$ . If all of the particles are moving with a Maxwell velocity distribution, then the average relative speed between two particles is greater than either one's average speed, so the collisions experienced by a particle are more frequent, and the mean free path of an individual particle turns out to be:

$$\lambda = \frac{1}{\sqrt{2} n \sigma_S} \quad (\text{B5})$$

Where  $\sigma_S$  is the total cross section for a collision (scattering) of the particle with another particle, which should be  $\sim \pi D^2$ , where  $D$  is the mean diameter of the two particles,  $\sim 1-3\text{\AA}$  for the atoms and molecules of the gas. The mean distance the particle will travel between collisions is

$$\bar{r} = \int_0^{\infty} r p(r) dr = - \int_0^{\infty} r \frac{d}{dr} P(r' > r) dr = \int_0^{\infty} \frac{r}{\lambda} \exp\left(-\frac{r}{\lambda}\right) dr = \lambda$$

thus the identification of  $\lambda$  with the mean free path. The mean time between collisions is:

$$\tau_S = \overline{\lambda / v} = (8/3\pi) \lambda / \bar{v} \approx \lambda / \bar{v} \quad (\text{B6})$$

## MOTION OF IONS; MOBILITY

An ionized atom of the gas will generally fly about randomly as described above, although its charge will cause an extra attractive force as it approaches a neutral atom, because of an induced dipole moment in the neutral atom. This means that the cross section for collision,  $\sigma_S$ , will be somewhat larger for an ion than for a neutral atom.

If an external electric field is imposed on the gas, the ion will experience a force which will accelerate it between collisions. As a consequence, the ion's average position will drift along the lines of electric force. If the mean time between collisions is short and the electric field is not too strong, then the additional velocity the ion acquires between collisions because of the electric force will be small compared to its average random

thermal speed, and each collision will be violent enough to completely randomize its velocity vector.

The ion will feel a force  $\vec{F} = q\vec{E} = m\vec{a}$  between collisions, and for any typical ion in the gas, its last collision will have occurred an average of  $\tau_s$  previously. Since an ion's average velocity immediately following a collision is 0, the average drift velocity of the ion in the gas will be  $\vec{a}\tau_s$ , so, using (B3), (B5), and (B6):

$$\vec{v}_d = \vec{a}\tau_s = \frac{q\vec{E}}{m}\tau_s \equiv \mu\vec{E} \quad (\text{B7})$$

$$\mu = \frac{q\tau_s}{m} = \frac{2}{3\sqrt{\pi}} \frac{1}{n\sqrt{m kT}} \frac{1}{\sigma_s} q \quad (\text{B8})$$

The following table lists some experimentally-determined mobilities of ions in neon gas.

Table B-1  
Mobilities in Ne gas at 760 Torr and 25°C

Ion	$\mu$ (cm <sup>2</sup> /sec/Volt)
Ne <sup>+</sup>	4.10
Ne <sup>++</sup>	5.85
Ne <sub>2</sub> <sup>+</sup>	6.11
Br <sup>+</sup>	6.95

### EINSTEIN RELATION BETWEEN MOBILITY AND DIFFUSION

Finally, let's use a simple statistical argument to determine the relationship between the mobility and the *diffusion coefficient*, or *diffusivity*, first articulated by A. Einstein in a paper of 1905.

Assume a gas containing some identical positive ions is subject to a constant, uniform electric field in the  $\hat{x}$  direction,  $\vec{E} = E_x \hat{x}$ . The potential energy of an ion in the field is then:  $q\vec{E} = -\nabla U \Rightarrow U(x) = -qE_x x$ . But for a gas in thermal equilibrium, we know that the probability of finding an ion with potential energy  $U$  is  $\propto \exp(-U/kT)$ , and the variation with position in the number density of ions in the gas must reflect this probability. Thus at thermal equilibrium it must be true that:

$$n_{ion}(x) \propto e^{-U(x)/kT} = e^{qE_x x/kT} \quad (\text{B9})$$

The drift velocity of the ions due to the force of the electric field is given by the mobility equation (B7); the total flux of ions (ions/area/time) due to the field will then be:

$$\vec{\Phi}_E = n_{ion}(x) \vec{v}_d = n_{ion}(x) \mu E_x \hat{x} \quad (B10)$$

On the other hand, an ion density gradient will result in diffusion of the ions away from areas of high density because of their random, thermal motion. This diffusion from regions of high density toward low density is described by the *mass diffusion equation*:

$$\begin{aligned} \vec{\Phi}_{diff} &= -D \nabla n_{ion} \\ \vec{\Phi}_{diff} &= -\hat{x} D \frac{d}{dx} n_{ion}(x) = -\hat{x} D \frac{qE_x}{kT} n_{ion}(x) \end{aligned} \quad (B11)$$

Where  $D$  is called the *diffusion coefficient* or *diffusivity* of the ions in the gas and has units of *length<sup>2</sup> / time*. In thermal equilibrium the density of the ions must be stable, so the total ion flux due to both the electric drift and diffusion at any point must vanish. Summing (B10) and (B11) gives the total flux; setting it to 0:

$$\begin{aligned} \vec{\Phi}_E + \vec{\Phi}_{diff} &= n_{ion}(x) \left( \mu E_x - D \frac{qE_x}{kT} \right) \hat{x} = 0 \\ \boxed{\frac{\mu}{q} = \frac{D}{kT}} & \end{aligned} \quad (B12)$$

Equation (B12) is the *Einstein relation* we were seeking. If we use our formula (B8) for the mobility we can derive an expression for the diffusivity,  $D$ :

$$\begin{aligned} D &= \frac{\mu}{q} kT = \frac{\tau_s}{m} kT = \frac{8}{3\pi} \frac{\lambda}{\bar{v}} \frac{kT}{m} \\ \therefore D &= \frac{1}{3} \lambda \bar{v} \end{aligned} \quad (B13)$$

where we've used the formula (B3) for  $\bar{v}$ . Equation (B13) for the diffusivity may also be derived directly from the kinetic theory of a classical, ideal gas. Note that diffusion will also tend to disperse the positive ion sheath as it moves outward from the center wire. This additional effect is not included in the discussion of the dynamics of the ion sheath in Appendix A to the experiment.



## APPENDIX C

## OUTPUT SIGNAL FORMATION IN THE GEIGER COUNTER CIRCUITRY

The discussion in this experiment's Appendix A concerned the flow of charge within the gas of the Geiger-Müller detector and the resultant flow of induced charge to the detector's center wire (anode) and outer tube (cathode). This appendix continues that discussion by describing how the induced charge flow through the external circuitry is used to generate an output voltage, and how this circuitry in turn affects the potential difference between the detector's anode and cathode.

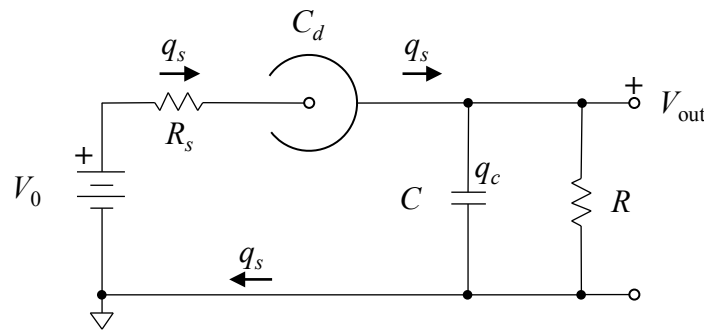


Figure C-1: A schematic diagram of the generic Geiger counter system analyzed here. The induced charge transferred through the circuit caused by charge motion within the detector is  $q_s$ ;  $q_c$  is the charge on the external circuit capacitance,  $C$ . The detector's capacitance is  $C_d$ . The high-voltage power supply voltage is  $V_0$ , whereas the measured output voltage is  $V_{out}$ . The voltage across the detector is  $V$ , as in equation (A2).

The circuit to be analyzed is shown in Figure C-1. The induced charge flow through this circuit caused by free electron and positive ion motion within the detector will generate an output voltage,  $V_{out}$ , across the circuit's parallel  $RC$  pair, which is in series with the detector's cathode. The high-voltage power supply,  $V_0$ , is connected as shown. The current-limiting resistor in series with the supply output,  $R_s$ , protects the user from serious electric shocks but otherwise has little effect on the circuit performance.

The voltage across the detector is  $V$ , which was expressed in terms of the charge distribution within the detector in equation (A2). Using the notation of Appendix A, the induced charge transferred through the circuit by the detector,  $q_s$ , is given by:

$$q_s(t) = q(t) - q_0 \quad (C1)$$

Depending on the value of  $R$ , some or most of this charge will wind up on the external capacitance,  $C$ . If the charge on  $C$  is  $q_c$ , then the output voltage is:

$$V_{out} = q_c / C \quad (C2)$$

Recall from (A4) the calculation of the detector capacitance,  $C_d \approx 2.6$  pF. Because its value is so tiny, we can assume that  $C \gg C_d$ , even if we work hard to make the external capacitance  $C$  small in an actual circuit. Finally, the current flow through the detector is  $dq_s / dt$ , the rate that the induced charge flows onto the center wire. Thus we know that:

$$V = V_0 - V_{\text{out}} - R_S dq_s / dt \approx V_0 - V_{\text{out}} \quad (\text{C3})$$

With these initial observations in mind, we may proceed with the detailed analysis. There are two distinct operating modes of the circuit, depending on the magnitude of the  $RC$  time constant of the circuit's components,  $\tau$ . If  $\tau$  is long compared to the time it takes for the positive ions to drift to the cathode, then the circuit is operating in *Charge Integration Mode*. Otherwise the circuit is operating in *Pulse Mode*. The circuit behavior is easier to analyze for the charge integration mode, so this operating mode will be investigated first.

### CHARGE INTEGRATION (CI) MODE

In this mode both  $R$  and  $C$  are relatively large, and  $\tau = RC \gg t_D + t_R$ , the time it takes for the ions in the detector to be collected at the cathode (outside tube) following a high-energy particle event (the *dead time* + the *recovery time*, as defined in the main text). In this case, the only purpose of  $R$  is to slowly drain off the charge on  $C$  and return  $V_{\text{out}}$  to 0, resetting the state of the system in preparation for detection of another event. Therefore the analysis proceeds assuming  $R \rightarrow \infty$ , and the current flow through it during a detection event is negligible. In this case  $q_c = q_s$ , because all of the induced charge must pass through the capacitor  $C$ , charging it (see Figure C-2 for an example output).

The voltage  $V(t)$  calculated from (C4) appears across the detector, and it must be used to replace  $V_0$  in equation (A5). Using the definition of  $q_s$  in (C1):

$$V(t) = V_0 - V_{\text{out}}(t) = V_0 - q_s(t)/C \quad (\text{C4})$$

$$V_0 - \frac{q_s}{C} = V = \frac{1}{C_d} \left[ q_0 + q_s - q_p \frac{\ln(r_s/r_e)}{\ln(b/a)} \right] \quad (\text{C5})$$

$$q_s \left( 1 + \frac{C_d}{C} \right) = q_p \frac{\ln(r_s/r_e)}{\ln(b/a)}$$

$$\boxed{(C + C_d)V_{\text{out}} = q_p \frac{\ln(r_s/r_e)}{\ln(b/a)}; V_p \equiv (V_{\text{out}})_{\text{max}} = \frac{q_p}{C + C_d}} \quad (\text{C6})$$

(charge integration mode)

So  $V_p$ , the maximum output voltage in this mode, is a direct measure of the charge in the ion sheath,  $q_p$ , and  $V_{\text{out}}(t)$  may be interpreted as a direct measure of the time evolution of the average ion sheath radius,  $r_s$ . Because  $C_d \ll C$ , it may be safely ignored in (C6).

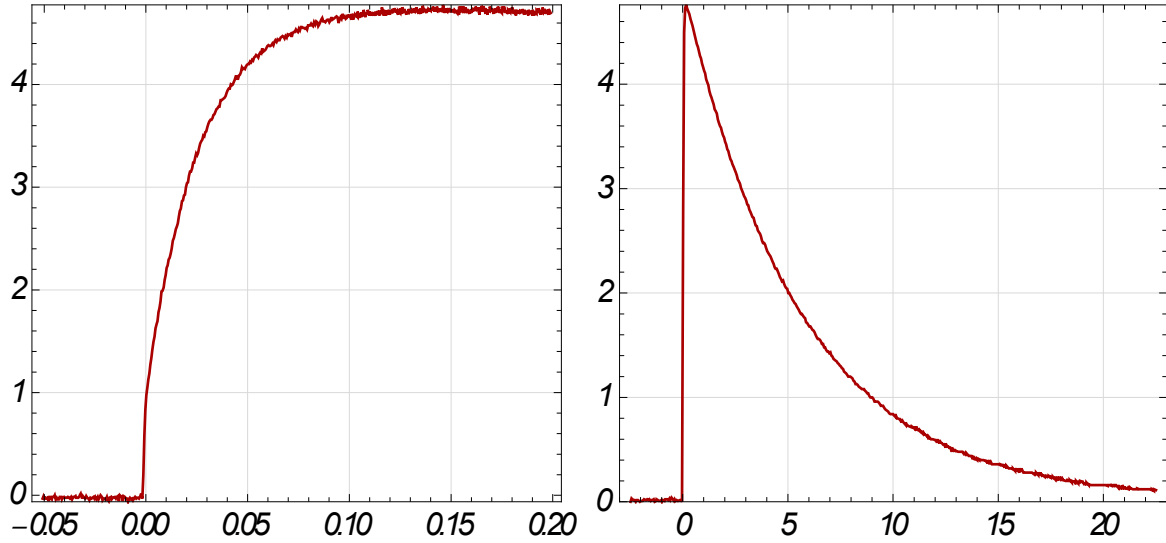


Figure C-2: Actual measured  $V_{\text{out}}$  for a typical detection in *Charge Integration Mode*. The vertical scale is Volts, and the horizontal scales are milliseconds. The left plot shows the integration of the induced charge up to its maximum,  $q_p$ . The right plot shows a similar pulse over a much longer time scale. Note the slow recovery of the circuit as  $C$  is discharged through  $R$ . Data plotted using *CurveFit*.

In the example  $V_{\text{out}}$  data of Figure C-2,  $V_p$  is 4.7 Volts. With  $C = 1.1 \text{ nF}$ , the ion sheath charge  $q_p = CV_p = 5.2 \times 10^{-9} \text{ Coulomb} \sim 3 \times 10^{10} e$ , and it took about 150 microseconds for all ions to be collected at the detector cathode. From the plot on the right, the  $RC$  time constant  $\tau \approx 6 \text{ ms}$ , about 40 times longer than the ion collection time.

Figure 3 in the main text gives another two examples of  $V_{\text{out}}$  data for CI mode. The right-hand image is a close-up of the first several microseconds of the output pulses. The sudden change in slope of each pulse may be interpreted as the point where ion sheath formation is complete, and the electron sheath has moved to the center wire's surface. The voltage at this point (where the cursors are positioned for the two pulses in the figure) corresponds to  $q_e$  in equations (A6) and subsequent in Appendix A. Thus we may define the output voltage  $V_e$ :

$$\boxed{(C + C_d) V_e \equiv q_e = q_p \frac{\ln(r_{s0}/a)}{\ln(b/a)}} \quad (\text{C7})$$

Let us rewrite the boxed equations (A7), (A13), and (A14) in terms of the capacitances and voltages (where we approximate  $C + C_d \rightarrow C$ ):

$$\boxed{r_{s0} = a (b/a)^{V_e/V_p}} \quad (C8)$$

$$\boxed{\mu = \frac{[r_{s0} \ln(b/a)]^2}{V_p \left[ V_0 - \frac{C}{C_d} \left( \frac{1}{2} V_p - V_e \right) \right]} \frac{dV_{\text{out}}}{dt} \Big|_{V_{\text{out}}=V_e+}} \quad (C9)$$

Equation (C9) shows how to estimate the *mobility*,  $\mu$ , from the slope of the output pulse just above the change in the slope at  $V_e$  (refer again to Figure 3 in the main text).

### PULSE MODE

In this mode  $R$  and  $C$  are chosen so that  $\tau = RC \ll t_D$ , so that the dead time  $t_D$ , may be determined (as in Figure 5 of the main text). The resistor  $R$  is small enough that the current through it may not be ignored, and not all of the induced charge from the detector will wind up on  $C$ . In this mode the external capacitance  $C$  may be just the stray capacitance of the circuit, but it is usually still larger than  $C_d$  by an order of magnitude or more. It also may be the case that the voltage drop across  $R_S$  is no longer negligible as well. If  $R_S \sim R$  or smaller, however, it turns out that its impact on the circuit behavior is small, since it enters the final equation only in terms  $\propto (R_S/R)(C_d/C) \ll 1$ . The derivation of the equation for  $V_{\text{out}}(t)$  is tedious and will be omitted. The result is (where terms of order  $C_d/C$  or smaller have been dropped):

$$\boxed{\frac{d}{dt} V_{\text{out}} + \frac{1}{\tau} V_{\text{out}} = \frac{q_p}{C} \frac{1}{\ln(b/a)} \left( \frac{1}{r_s} \frac{dr_s}{dt} - \frac{1}{r_e} \frac{dr_e}{dt} \right)} \quad (\text{pulse mode}) \quad (C10)$$

Equation (C10) has the same structure as that of a simple, one-pole, RC high-pass filter. To interpret the equation, first note that the second term on the left-hand side is just the negative of the discharge rate of  $V_{\text{out}}$  through  $R$ , so that this side is simply the difference between the actual rate of change of  $V_{\text{out}}$  and the rate expected because  $\tau$  is finite. Second, note that the leading  $q_p/C$  is just the maximum expected  $V_{\text{out}}$  if this were a CI mode (cf. equation (C6)). Thus whenever  $(d/dt)V_{\text{out}} \gg V_{\text{out}}/\tau$ , the output is the same as in CI mode (using the current value of  $C$ !). When the derivatives on the right-hand side of (C10) are approximately constant for a while, however, then so will be  $V_{\text{out}}$ . In this case:

$$V_{\text{out}} = R q_p \frac{1}{\ln(b/a)} \left( \frac{1}{r_s} \frac{dr_s}{dt} - \frac{1}{r_e} \frac{dr_e}{dt} \right), \quad (\text{if } V_{\text{out}} \approx \text{constant}) \quad (C11)$$

Lastly, as the positive ions move outward toward  $b$ , the right-hand side of (C10) eventually becomes small, and  $V_{\text{out}}$  then discharges toward 0 with time constant  $\tau$ .

The choice of  $\tau$  determines the maximum event detection rate the Geiger counter system can handle without pulse “pile-up,” where  $V_{\text{out}}$  responds to another event before fully discharging to 0. Since the Geiger-Müller detector suffers from *dead time*, as described in the main text (Figures 3 and 4), it is appropriate to choose  $\tau < t_D$ . As mentioned above, the circuit you will test has ,  $\tau \ll t_D$  so the dead time  $t_D$  may be accurately measured.

