Data Analysis Pages

Your lab notes for an experiment are concluded by several pages of analysis of your data and comparison to theory. A final summary and conclusions page completes the write-up. 40% of your grade for an experiment will be based on this effort.

Look for the physics, and evaluate theories quantitatively and critically. Use the experiments to answer or pose interesting questions! Use the data to determine the values of fundamental constants of nature (with uncertainties). Don't leave a wiggle on a plot unnoticed if it is more than just noise.

Unlike most research lab notebook records, you will include a section in your lab notes for an experiment which provides a thorough, formal analysis of your data and uncertainties, compares your results to the theory being tested, and, if relevant, determines the values of the theory's free parameters along with uncertainties. You will perform this analysis and record it in you lab notebook rather than generate a formal paper or lab report of your results.

Your analysis should be much more detailed and thorough than what would be found in a typical research paper in a scientific journal. In the interests of brevity, most journal articles leave out many of the actual calculations and judgements of the experimenter. The data presented are often a specially-selected subset of the full data set to particularly reinforce the writers' conclusions. This "cherry picking" of the data has no place in your lab notebook!

All results should include an error analysis at the level of *Physical Data Analysis*. Use the *CurveFit* package for *Mathematica* for data analyses. Include residual plots of fits versus data. Use χ-squared calculations in your analyses. Propagate errors (uncertainties).

All graphs and their axes must be labeled. Clearly differentiate real data from theoretical curves or fits. Always write down the formulas you use, and *always include units* with numerical quantities. Numerical results derived from measured data should always include uncertainties.

Clearly differentiate the effects of possible systematic error on your results from errors introduced by noise. If you performed a calibration of part of the apparatus, clearly show how the calibration was performed and how it factors into your results and their uncertainties.

Clearly highlight important findings and conclusions.

An example of analysis lab notebook pages

The following several pages show one way of presenting a portion of the analysis of an experiment. In this case, it is an analysis of the behavior of the input signal level to the RLC resonant circuit of Experiment 2.

> This is an example of an analysis which goes beyond the basic requirements of the experiment, which is important if you want to get a very good grade for the course.

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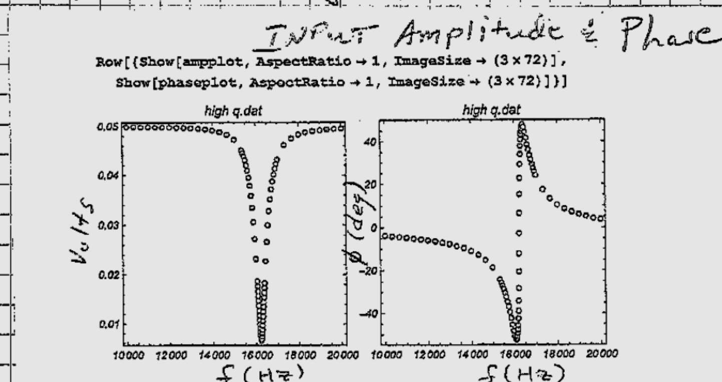
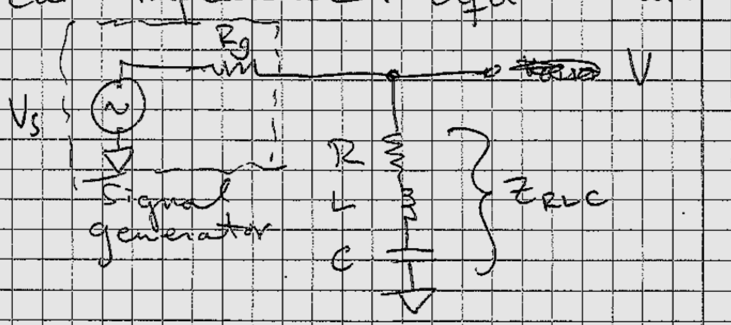
Analyzing the shape of the input voltage v. freq response in Exp 2.

The input Amplitude & Phase v freq (high-Q config):



Theory: The input amplitude dips at resonance because the 50Ω signal generator output resistance becomes large compared to the circuit impedance. Equiv circuit:



Voltage divider: $\frac{V}{V_s} = \frac{Z_{RLC}}{R_g + Z_{RLC}}$

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Test this hypothesis (Theory)

$$\frac{V}{V_S} = \frac{Z_{RLC}}{R_g + Z_{RLC}}; \quad Z_{RLC} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Notes equ (4)

with $Z_0 = \omega_0 L = \frac{1}{\omega_0 C}$ Notes equ (5)

$Q = \frac{Z_0}{R}$ " (6)

and define $\boxed{\frac{R_g}{R} = \varphi}$

$$\boxed{\frac{V}{V_S} = \frac{1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}{\varphi + 1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}}$$

f_0 : res freq

Model 1

$$\boxed{\therefore \left|\frac{V}{V_S}\right| = \left(\frac{1 + y^2}{(\varphi+1)^2 + y^2}\right)^{1/2} \quad y \equiv Q\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$

Will use fitandfunction notebook with Curvefit to fit this model; results on next page.

with $\varphi = 6.66$, and if $R_g = 50\Omega$,

then $\boxed{R = 7.5\Omega}$

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Y[Q_, f0_, f_] := Q (f / f0 - f0 / f)

FitData[Vs Sqrt[(1 + Y[Q, f0, freq]^2) / ((1 + ρ)^2 + Y[Q, f0, freq]^2)], freq, {Vs, Q, f0, ρ}, {.05, 140, 16220, 6}]

n = 93
p = 4
```

$$f[freq] = Vs\sqrt{\frac{1+\left(-\frac{f0}{freq}+\frac{freq}{f0}\right)^2 Q^2}{\left(-\frac{f0}{freq}+\frac{freq}{f0}\right)^2 Q^2+(1+\rho)^2}}$$

Fit of (x,y) (unweighted):

Vs	=	0.0500016	±	8.12586×10^{-1}
Q	=	138.735	±	0.277049
f0	=	16223.4	±	0.0150876
ρ	=	6.66856	±	0.0230739

Std. Deviation = 0.0000398743

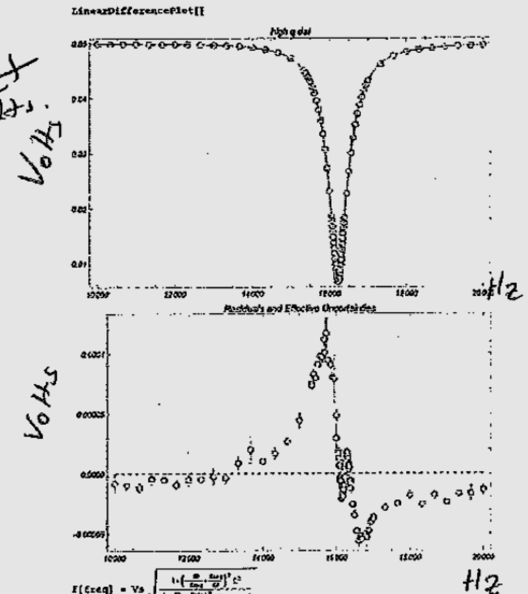
Q and f₀ consistent with resonance gain results.

Fit of (x,y±σy):

Vs	=	0.0500002	±	4.90039×10^{-7}
Q	=	138.425	±	0.0161743
f0	=	16223.2	±	0.00316256
ρ	=	6.65799	±	0.000715363

$\frac{\chi^2}{n-p} = 275.674$

{Vs → 0.0500002, Q → 138.425, f0 → 16223.2, ρ → 6.65799}



$$f[freq] = Vs\sqrt{\frac{1+\left(-\frac{f0}{freq}+\frac{freq}{f0}\right)^2 Q^2}{\left(-\frac{f0}{freq}+\frac{freq}{f0}\right)^2 Q^2+(1+\rho)^2}}$$

Vs	=	0.0500002	±	4.90039×10^{-7}
Q	=	138.425	±	0.0161743
f0	=	16223.2	±	0.00316256
ρ	=	6.65799	±	0.000715363

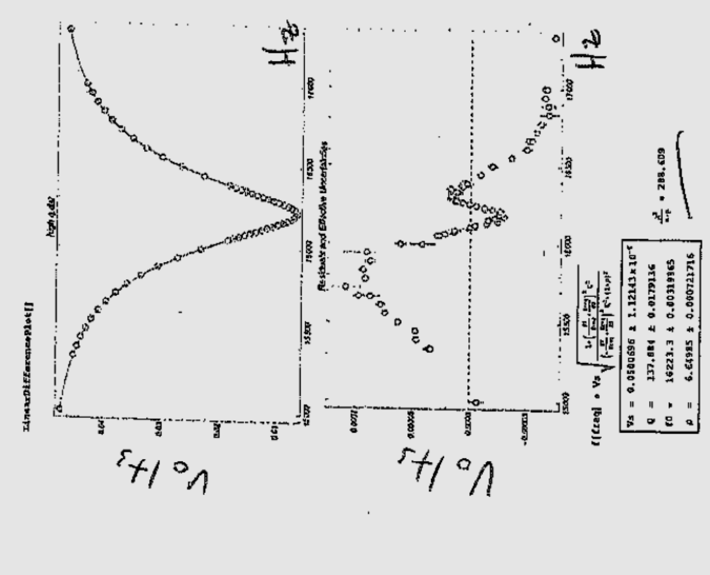
$\frac{\chi^2}{n-p} = 275.674$

As with resonance gain data, significant structure to residuals implies that the simple theory (Z_{RLC}) is incomplete.

Note that although theory is inadequate to explain ~~fwd~~ fine detailed structure to the shape of the resonant dip, it very adequately describes the overall shape to $\sim \pm 10^{-4}$ V out of ~~5×10~~ 5×10^{-2} V $= V_s$ (better than 1%)

Comments: The detailed structure of the Shape of the resonance is reminiscent of what was seen in the gain magnitude fits. Fitting a data subset closer to the resonance doesn't remove this discrepancy (plot ~~next page~~)

This may indicate that the inductor's impedance vs. freq is significantly more complicated than our simple model.



Fit to input voltage data for freq in 14.9 kHz – 17.5 kHz.

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Fitting the phase.

$$\frac{V}{V_S} = \frac{1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}{\varphi + 1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$

$$\phi = \tan^{-1}\left(\frac{Im}{Re}\right) = \cot^{-1}\left(\frac{Re}{Im}\right)$$

multiply top & bottom by conj of denominator;

$$\frac{V}{V_S} = \frac{\left[1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right]\left[1 + \varphi - jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)\right]}{|\text{denom}|^2 \leftarrow \text{real.}}$$

ratio of $\frac{Im}{Re} = \left(\frac{Im}{Re}\right)_{\text{numerator}}$

$$Re = \varphi + 1^2 + Q^2\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2; \quad Im = \varphi Q\left(\frac{f}{f_0} - \frac{f_0}{f}\right)$$

Use 2-argument ArcTan function so result is in the correct Quadrant. ~~Divide by~~
Convert result to degrees

First fit shows a clear phase offset and maybe a linear phase variation (slope) to the residuals. Thus will include this extra (systematic?) error correction to the fitting function:

Theory

$$\boxed{\phi(f) = \theta + \text{slope}(f - f_0) + \tan^{-1}\left(\frac{\varphi Q\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}{\varphi + 1 + Q^2\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2}\right)}$$

Results next page.

```
FitData[ArcTan[{1 + r + Y[Q, f0, freq]^2}, r Y[Q, f0, freq]] 180 / Pi + theta + slope (freq - f0),
 freq, {r, Q, f0, theta, slope}, {0, 140, 16000, 0, 0}]

n = 93
p = 5
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$$f[\text{freq}] = (-f0 + \text{freq})\,\text{slope} + \text{theta} + \frac{180\,\text{ArcTan}\left[1 + \left(-\frac{f0}{\text{freq}} + \frac{\text{freq}}{f0}\right)^2 Q^2 + r,\ \left(-\frac{f0}{\text{freq}} + \frac{\text{freq}}{f0}\right) Q\, r\right]}{\pi}$$

Fit of (x,y) (unweighted):

r	=	6.67391 ± 0.00463597
Q	=	138.472 ± 0.0974919
f0	=	16223.1 ± 0.0365879
theta	=	-2.12657 ± 0.00967977
slope	=	$-0.000180316 \pm 5.0404 \times 10^{-5}$

Std. Deviation = 0.0825727

Fit of $(x, y \pm \sigma_y)$:

r	=	6.67763 ± 0.00175366
Q	=	138.45 ± 0.0442581
f0	=	16223. ± 0.0242885
theta	=	-2.1462 ± 0.00337560
slope	=	$-0.000187943 \pm 1.60537 \times 10^{-4}$

$\frac{\chi^2}{n-p} = 6.6275$

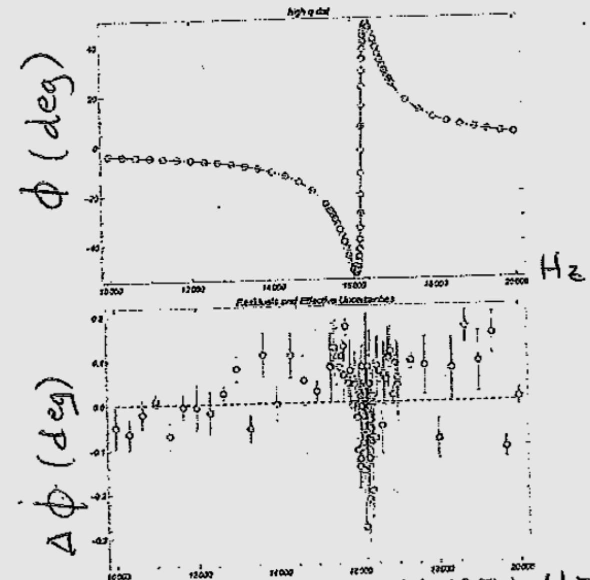
{r → 6.67763, Q → 138.45, f0 → 16223., theta → -2.1462, slope → -0.000187943}

As with gain phase fits, model fits very well (on a linear ϕ v. freq background)

z1

Input Voltage Phase

LinearDifferencePlot[]



$$f[\text{freq}] = (-f0 + \text{freq})\,\text{slope} + \text{theta} + \frac{180\,\text{ArcTan}\left[1 + \left(-\frac{f0}{\text{freq}} + \frac{\text{freq}}{f0}\right)^2 Q^2 + r,\ \left(-\frac{f0}{\text{freq}} + \frac{\text{freq}}{f0}\right) Q\, r\right]}{\pi}$$

r	=	6.67763 ± 0.00175366
Q	=	138.45 ± 0.0442581
f0	=	16223. ± 0.0242885
theta	=	-2.1462 ± 0.00337568
slope	=	$-0.000187943 \pm 1.60537 \times 10^{-5}$

$\frac{\chi^2}{n-p} = 6.6275$

Comments:

(1) f_0 and Q are very consistent with Voltage magnitude results:

fit	f_0	Q
Mag	16.223 KHz	138.43
Phase	16.223 KHz	138.45

} both match to about a part in 10^4. Note that f_0 uncertainties from the fits are much smaller, thoug

(2) the linear ϕ "background" could be explained by a constant time delay t_d between sync signal and generator sine wave output: $t_d\omega = \phi \rightarrow t_d = \frac{d\phi}{d\omega} = \frac{1}{2\pi}\frac{d}{}$

thus: $t_d = \frac{1}{2\pi \frac{rad}{Hz}}\left(-1.9\times10^{-4}\frac{deg}{Hz}\right)\left(\frac{\pi\ rad}{180\ deg}\right) = 0.52\,\mu s.$

uncertainty: $\frac{\sigma_{slope}}{slope} = \frac{1.6\times10^{-6}}{1.9\times10^{-4}} = 0.8\,\%$

$$\boxed{t_d = 0.52\,\mu s \pm 0.8\%}$$

(3) Simple theory quite accurate, except for fine structure in amplitude data at the resonance (at the 0.1 mV level).