Experiment 7

THE KELVIN ABSOLUTE VOLTMETER
and the speed of light

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INTRODUCTION

The force between two electric charges described by Coulomb's law is an aspect of one of the fundamental forces of nature, the electromagnetic force. By accurately measuring this force law one may determine not only this electrostatic aspect of electromagnetism, but also determine the speed of propagation of electromagnetic waves, the speed of light! This possibly surprising result follows from the classical theory of electromagnetism as described by Maxwell's equations.

The Maxwell equations in a linear medium without free charges or currents are (in SI units):

$$\begin{aligned} \nabla \cdot \vec{E} = 0 \qquad & \nabla \times \vec{E} = -\partial \vec{B} \big/ \partial t \\ \nabla \cdot \vec{B} = 0 \qquad & \nabla \times \vec{B} = \mu\varepsilon \;\; \partial \vec{E} \big/ \partial t \end{aligned} \tag{1}$$

where μ and ε are the permeability and permittivity of the medium. In a vacuum, $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$. In air, we still have $\mu = \mu_0$, and $(\varepsilon - \varepsilon_0)/\varepsilon_0 \sim 6\times10^{-4}$, so air is a good approximation of vacuum for this experiment. Taking the curl of each of the right-hand equations in (1) and using the vector identity $\nabla\times\nabla\times\vec{a} = \nabla(\nabla\cdot\vec{a}) - \nabla^2\vec{a}$ along with the vanishing divergences in (1) lead to identical wave equations for the electric and magnetic fields in vacuum:

$$\begin{aligned} \nabla^2 \vec{E} = \mu_0\varepsilon_0 \;\; \partial^2 \vec{E} \big/ \partial t^2 \\ \nabla^2 \vec{B} = \mu_0\varepsilon_0 \;\; \partial^2 \vec{B} \big/ \partial t^2 \end{aligned} \tag{2}$$

Clearly, $\mu_0\varepsilon_0$ has the dimensions of (velocity)$^{-2}$, and this velocity of course turns out to be the phase velocity of the wave, which is the speed of light. Since $\mu_0 = 4\pi\times10^{-7}$ Volt sec^2 Coulomb^{-1} meter^{-1} *exactly* (the 4π comes from the area of a unit sphere, and the 10^{-7} Volt sec^2 Coulomb^{-1} meter^{-1} just provides a units conversion factor), a measurement of ε_0 is also a measurement of the speed of light, *c*.

In this experiment you will determine the potential difference required to generate a given force between two parallel charged plates. Using your data you will then determine a value for ε_0, and, consequently, a value for *c*. To establish an accurately-known force between the plates, one of them is attached to an arm of a sensitive analytic balance (see figure 1 on the next page for a photo of a typical setup).

THEORY

Consider two infinite parallel planes separated by a distance d and carrying uniform surface charge densities of $\pm\sigma$. The electric field produced by either plane would then be everywhere perpendicular to it and uniform throughout space (except in the infinitesimal thickness of the plane itself) with magnitude $|E| = \frac{1}{2}|\sigma|\big/\varepsilon_0$, the fields on either side of the plane pointing away from or toward the plane depending on the sign of σ. The total electric field for the two-plane configuration would be the vector sum of the fields from the planes (because linear superposition

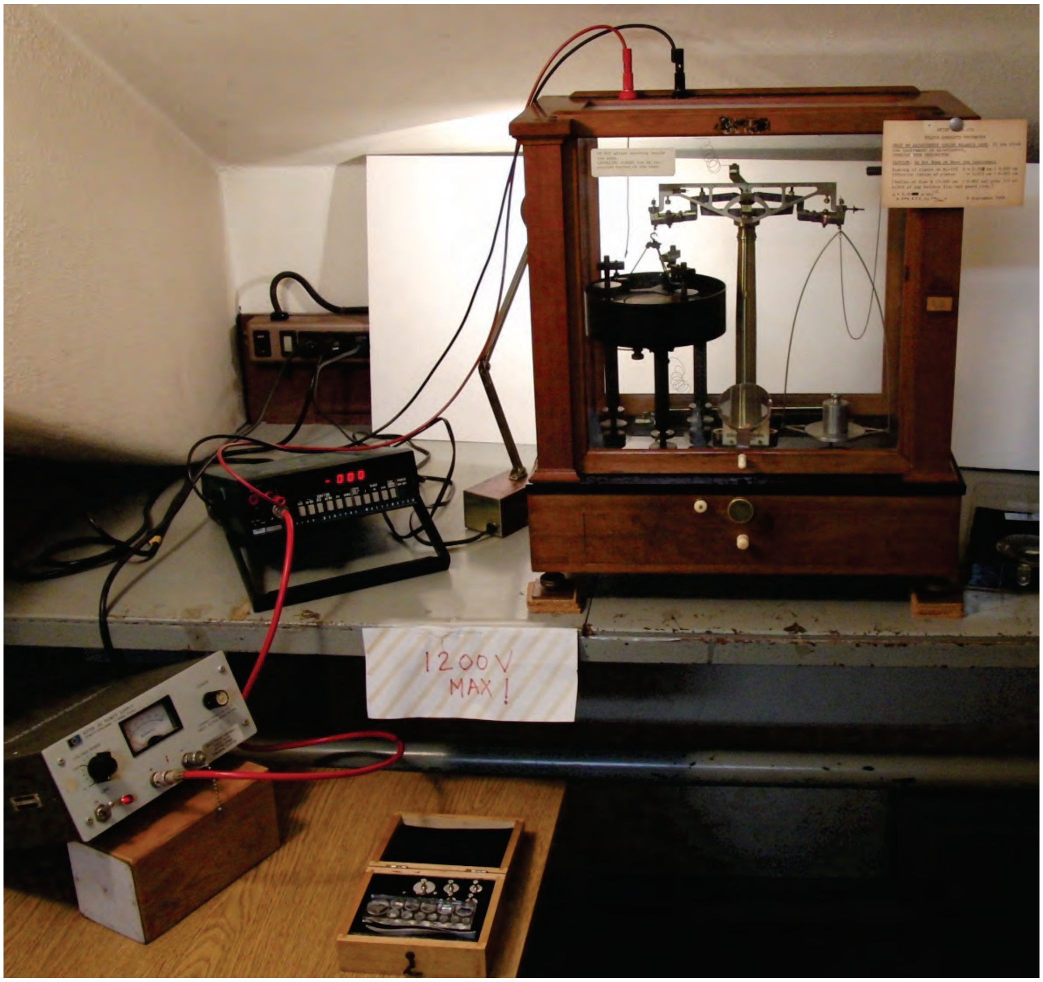


Figure 1: A setup for the determination of ε_0. Adding a small, precision mass to the right-hand pan of the analytic balance applies an accurately-determined force to the upper plate of a parallel-plate system. The high-voltage power supply at lower left charges the plates and maintains the potential difference between them, which is measured by the digital voltmeter next to the balance. When the potential difference is increased sufficiently, the Coulomb force between the plates will exceed that applied by the mass added to the balance, and the upper plate will fall toward the lower one. The masses added to the balance are only on the order of a few 10's of milligrams. The modifications to the balance for this experiment were designed and built by Dr. Wolfgang Panofsky in the 1940's.

holds for the fields). Thus the total field would be nonzero only between the two planes, where it would be given by equation (3):

$$E = \sigma / \varepsilon_0 \tag{3}$$

The potential difference between the two planes can be calculated by performing a line integral of the electric field along a line segment joining the two planes. Since the field is constant between the two planes and is perpendicular to them, the integral is trivial and results in:

$$V = Ed = \sigma d / \varepsilon_0 \tag{4}$$

The charge on each plane experiences a net force because of the electric field of the other plane (and *not* the total electric field due to both planes — why?). The electrostatic force per unit area (pressure) on either plane is thus:

$$F/A = \tfrac{1}{2}\sigma E \tag{5}$$

and is attractive (why?). In terms of the potential difference,

$$\boxed{F/A = \tfrac{1}{2}\varepsilon_0 V^2/d^2} \tag{6}$$

In this experiment you will use equation (6) to determine ε_0 from your data relating F and V.

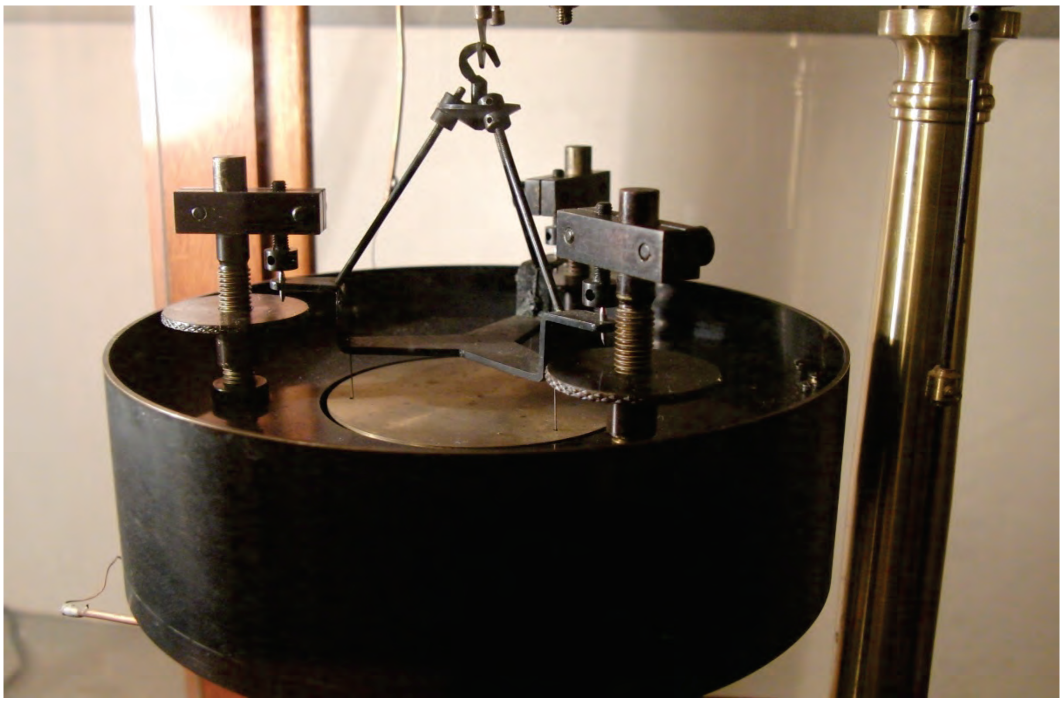


Figure 2: A close-up of the charged plate assembly. A central brass disk with a radius of approximately 3 cm is attached to one arm of the analytical balance whereas the rest of the assembly is fixed. The disk lies in the same plane with its surrounding "guard" ring. The guard ring is attached along its periphery to the black cylinder. The cylinder encloses a larger plate just below the disk and guard ring (not visible in the photo). This second, hidden plate is insulated from the surrounding cylinder and is electrically attached to a high-voltage power supply. All the visible metal parts are electrically attached to the earth ground terminal of the supply so that a potential difference may be established between the lower, hidden plate and the upper brass disk and its guard ring.

THE EXPERIMENTAL APPARATUS

The apparatus shown in figure 1 will allow you to accurately measure the relationship between the force on one of a pair of charged plates and their mutual potential difference. A close-up photo of the plate assembly is shown in figure 2 and a cross-sectional schematic of it is shown in figure 3 (next page). A conducting circular disk is positioned a short distance above a larger conductor, and a potential difference can be set up between the two conductors using a high-voltage power supply. The disk is surrounded by a conducting "guard" ring at the same potential, so that the upper conductor is effectively larger than the central disk. As a result, the lower surface of the disk acts as a small section of one of the two infinite charged planes discussed in the theory leading to equation (6).

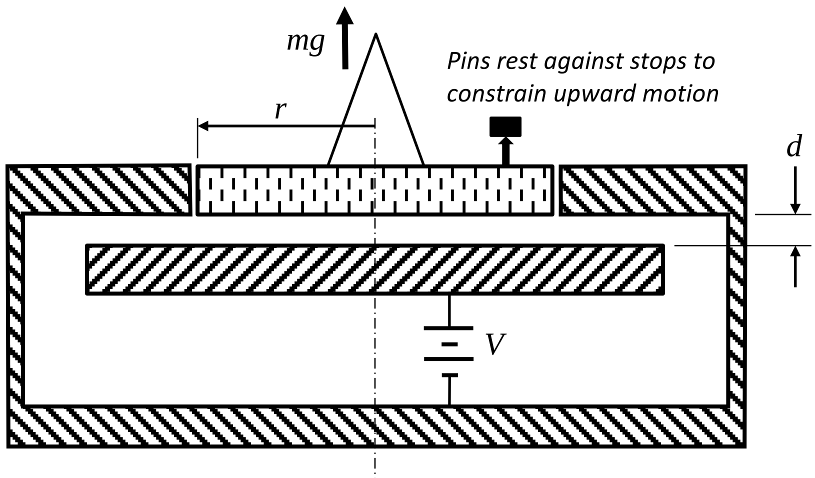


Figure 3: This cross section of the plate assembly illustrates the relative locations of the upper, movable plate (radius *r*), its guard ring, and the lower, fixed plate. At equilibrium the two plates are separated by distance *d*, approximately 0.3cm. A small mass *m* added to the other arm of the balance applies an upward force *mg* to the upper plate, but pins and stops (figure 2) prevent the plate from moving further upward. The power supply sets the potential difference *V* between the plates, which causes a downward force on the upper plate in accordance with equation (6). If this force exceeds *mg* the upper plate will accelerate downward toward the lower plate.

Equation (6) is only valid if edge effects of the conductors can be neglected, that is, if the electric field is everywhere normal to the two plates within the region near the movable disk. The guard ring and the large size of the lower plate ensure that the distortions of the field lines away from the vertical near the edges of the conductors are kept away from the area near the disk. The small gap between the guard ring and the disk does cause a slight field distortion, however. The correction for this effect is to assume that the moveable disk has an "effective" radius r_{eff} which includes ½ of the gap width. See if you can come up with a justification for this correction. Discuss this issue with your TA or the Laboratory Administrator if you still aren't convinced.

The apparatus you will use was designed and built by Wolfgang Panofsky (Caltech Ph.D. 1942) in the 1940's (using an analytical balance made by Christian Becker, Inc., sometime between 1915 and 1922). Dr. Panofsky was an important physicist who worked on the Manhattan project during the Second World War, discovered the π^0 meson (with Steinberger and Steller) in 1950, and became the first director of the Stanford Linear Accelerator Center (SLAC). He led the design and construction of SLAC, commissioned in 1966, then the world's most powerful accelerator (and most expensive physics laboratory). He directed SLAC until 1984, and during his tenure 3 Nobel prizes were awarded to researchers using the facility.

PROCEDURE AND ANALYSIS

Be careful, think about what you are doing, and don't be the first one to ruin the priceless, historic instrument you will be using!

The procedure is comprised of two major steps: (1) setup, level, and balance the apparatus; and (2) take voltage data for a variety of masses to determine the functional relationship between V and m.

At no point should you make any adjustment to the hardware inside the glass case containing the balance. If you are uncertain about the leveling and balance procedures, ask the laboratory administrator!

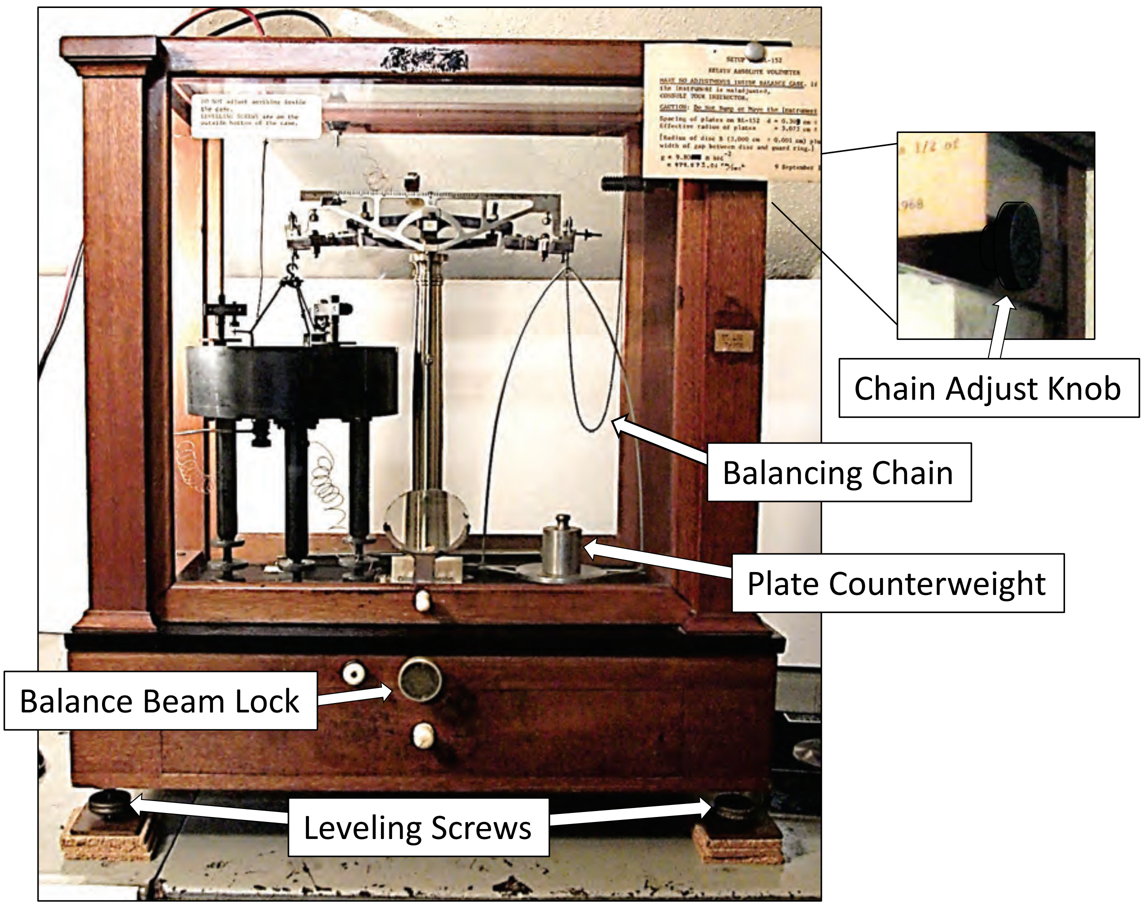
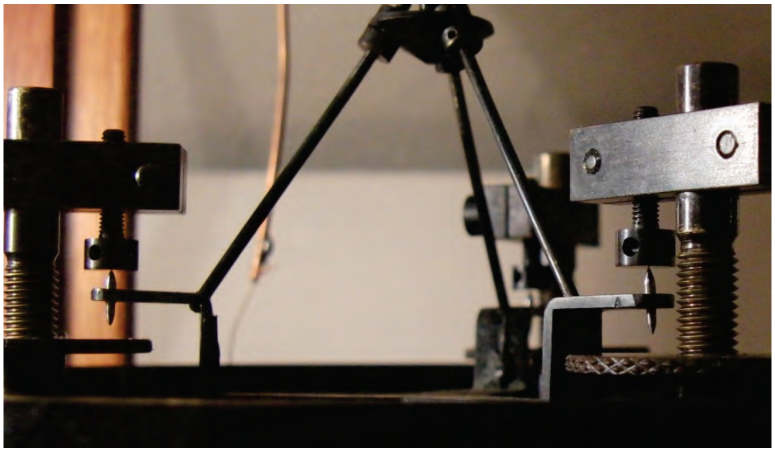


Figure 4: The controls you will use to level and balance the apparatus are all on the outside of the case: the Balance Beam Lock, Leveling Screws, and Chain Adjust Knob. Lock and unlock the balance cross-beam by turning the Balance Beam Lock knob, which will also recenter the fulcrum if you bump the beam. The plate counterweight must remain on the right pan because it balances the weight of the movable charged disk. The length of the balancing chain is adjusted by the knob near the top of the right side of the case (behind the calibration information card). Turning the knob makes precision adjustments to the zero-force balance point of the movable charged disk. ***Make no adjustments to anything inside the glass case containing the balance.***



↑Pins up (initial position) ↓Pins down ($F > mg$)

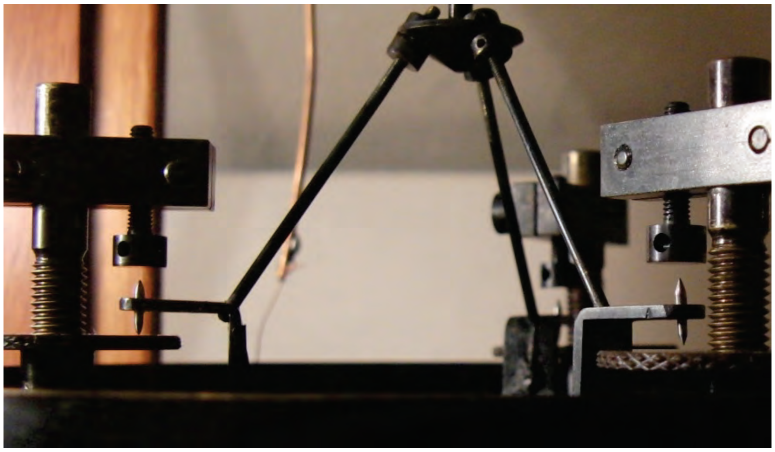


Figure 5: The alignment pins and stops constraining the motion of the circular charged plate. The top photo shows the plate in its proper initial position to begin a data point. If the voltage is 0 and no additional mass is added to the balance, then the proper balance point is where the 3 pins are each just touching their respective stops (the upside-down screws). After adding some small mass to the right-hand balance pan, you will slowly increase the voltage until the pins just start to move downward from their upper stops. The assembly should then continue to fall until the pins contact the lower stops as shown in the bottom photo.

To begin the setup of the apparatus, familiarize yourself with the balance and its parts by referring to figures 4 and 5 as you examine the instrument. The power supply should be turned on with the voltage set to zero, and the balance beam should be unlocked. Leveling is accomplished by carefully adjusting the level screws on the front two feet of the balance. Use the chain adjust knob to move the alignment pins toward and away from the upper stops (figure 5 top photo). The balance is leveled when all three pins make or break contact with the upper stops **simultaneously**. Only small adjustments of the leveling screws should be required to achieve this result. When properly leveled the movable disk will also be centered in its guard ring.

Don’t forget to copy down the apparatus specifications listed on the calibration card!

Once leveling is complete the balancing chain should be carefully adjusted so that the neutral position of the balance is where the three alignment pins are just touching the upper stops. It is in this position that the distance *d* between the charged plates matches the value on the calibration card. At this point the setup of the apparatus is complete.

If you subsequently change the chain length or the leveling screws, then the setup procedure must be repeated, and during analysis your subsequent data should not be combined with any previously recorded data (why?).

The masses you will add to the right-hand balance pan are of class "S-1" precision (National Bureau of Standards Handbook 77 Vol. III, p.644/26, available in the lab). They must be handled carefully using forceps (not your fingers!).

After adding a small mass *m* to the right-hand balance pan (the range of masses you can successfully use for the experiment is about 50 milligrams to 200 milligrams), you will find the voltage *V* which generates a force on the upper plate of just greater than *mg*, so that the upper plate starts to fall toward the lower one. As you approach the required voltage from below, you must slowly and carefully add one volt at a time while watching the alignment pins for signs of motion.

If you use a separate digital voltmeter to measure the applied voltage, then *do not exceed its voltage limit* (typically 1200V). This limit will determine the maximum mass you can use (approximately 200 milligrams for 1200V).

The rate you change the voltage can have a large impact on the systematic error you introduce into the experiment: add voltage too quickly, and you will have increased it too far before you observe that the pins are accelerating away from their stops; add voltage too slowly, and some random vibration of the apparatus will start the pins moving at too low a voltage. Because the force between the plates increases as they move toward each other, you will have to lower the voltage a few hundred volts to get the upper plate to rise to its starting position, so that you may repeat the data point.

You must repeat the data point for each mass value a few times in order to determine the uncertainty in the voltage determination. You should comment in your lab notebook about anything noteworthy that may have affected the accuracy or added systematic error to a particular data point, so that you have extra information to help you decide whether or not to include that data point in your analysis.

Acquire voltage data for many different mass values throughout the useful mass range. Organize yourself so that you cover the mass range completely, then go back and include additional mass values between your first set of choices. It might be useful to start with 100 mg, since you will work out the expected voltage for this mass in the prelab problems.

Data Analysis

For each mass value you should have several voltage data points. Clearly the distribution of these points will allow you to determine an uncertainty to assign to them. *CurveFit* provides a function (under the *Modify data points*: *Basic data manipulations* palette submenu) to determine the appropriate uncertainty to assign to Y values, if you have several Y values for any given X value. Since you used a variety of fixed masses and measured several voltages for each mass, it would be logical to create a text file of your data with data lines consisting of mass – voltage pairs. The *CurveFit* sample data file *Zeeman.dat* shows how to format such a file for it to be readable using the *CurveFit* Data I/O functions, although this file is of sample data for a different experiment.

Once you've created a data file (with only one mass – voltage pair per line, and with the two data values on each line separated by whitespace), load it into *CurveFit*. The mass should be X and the voltage Y; if not use the *Modify data points* palette menu to find the function to swap X and Y values. You may then use the *Analyze Y data and assign σy's* selection to calculate and assign uncertainties to your voltages.

Since your adjustment of the balancing chain was probably not perfect, there is a fixed, unknown mass offset which must be determined by the fit you choose; otherwise there could be a significant systematic error in your mass values. Clearly a linear fit of V^2 to mass m would include a mass offset as a fit parameter; the slope of the fit would provide a determination of ε_0. You can use *CurveFit*'s *Transform Data* function to square your voltages (and propagate errors).

Do your data support the claim that $V^2 \propto m$? To what level of accuracy? What is your experimental determination (with uncertainties) of ε_0 and the speed of light, c? Given the precision of the masses and that of the dimensions of the apparatus, which parameter is the major limiter to your experiment's accuracy?

PRELAB PROBLEMS

1. Derive equation (6) from the previous ones in its section.

2. Answer the two "why?" questions in the theory discussion on page 7-3.

3. After substituting *mg* for *F* and πr^2 for *A* in equation (6), rewrite it as $V^2 = k\,m$. What is the expression for the constant *k* in terms of ε_0 and the other parameters? Calculate the expected *V* for a mass *m* of 0.100 gm (100 milligrams), if $\varepsilon_0 = 8.854\times10^{-12}\ \text{Coulomb}/(\text{meter Volt})$. Assume the following for the specifications of the apparatus:

Plate spacing	$d = 0.309 \pm 0.001$ cm
Effective radius	$r = 3.073 \pm 0.001$ cm
Gravity	$g = 979.6\ \text{cm}/\text{sec}^2$

   Show that the units in your result work out to Volts, given the units provided for ε_0. What is the fractional (%) uncertainty in the constant *k* given the stated uncertainties in the specifications stated above (an exercise in error propagation)? What is the uncertainty in *V* which results? How does this compare with a fractional uncertainty of 10^{-4} in a precision mass you will use?

4. Provide a brief, verbal description of your experiment procedure to your TA.